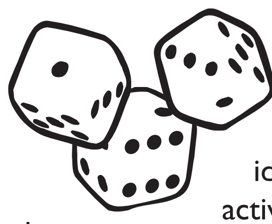
Dice Activities for Mathematical Thinking

Foreword

Successful math students manipulate numbers mentally. The activities in *Dice Activities for Mathematical Thinking* were created by teachers to engage students in developing fluency with the mathematical concepts of square numbers, square roots, prime numbers, factorials, summation, and integers. The activities are designed to empower students with the ability to address mathematical problems and challenges with a sense of curiosity and confidence.

These activities focus on the NCTM standards of Number and Operations and Algebra. They also address the standards of Reasoning, Problem Solving, and Probability. The NCTM standards are the framework of all published mathematics programs and state and local curricula frameworks; thus, these activities are easily integrated into a scope and sequence whenever the topic is addressed. In many instances the teacher will want to replace the activity in the school-based text with those found in this book, as they are challenging, more apt to provide long-term mastery, and develop a profound interest and curiosity about math.



The authors currently use *Dice Activities for Mathematical Thinking* as part of their curriculum to train elementary school teachers in how to teach mathematics. The activities require only the use of dice, a commonly available manipulative. They are easily adapted to home schooling and for parents who wish to participate in the mathematics education of their children. They provide an opportunity for students to play with big mathematic ideas without paper-and-pencil drill. The activities are engaging, generate a friendly competition, and provide immediate use of and reason for learning these mathematic concepts.

Our work is continually expanding, and we welcome any suggestions for modification of these activities that will lead to greater mathematical thinking on the part of our students. Submit any suggestions to: mathofcourse@gmail.com.

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Authors' Introduction

Too often teachers are under pressure to get through the curriculum, which is often determined by chapters in the adopted text programs, driven by pacing charts, and measured by district-imposed assessment instruments. The purpose of chapter tests is to ensure that all teachers are not only using the same program but are essentially on the same page. "Research based" is often the mantra for system adoption of programs and practices, but who is looking at what is happening to the students' mathematical knowledge base? While test scores may improve or at least remain stable, students quickly forget what they don't understand. Short-term memory serves well for passing the frequent tests, but unless students have a firm grip on the content and concepts, it all falls away and must be retaught annually. Teachers in subsequent years wonder why the students don't know what is expected of them. Certainly the human mind is capable of learning more efficiently. The problem must lie in how math is taught, and therefore learned, and not in a malfunction of the students' capabilities.

So how does this particular book address these issues? It is important to acknowledge the value of these dice activities for students and how they impact their mathematical competence. The most powerful tool that needs to be developed in the mathematical journey is each student's mathematical thinking skills. If a student develops the disposition towards thinking mathematically, employs a curiosity of about why, and wonders how, then the student is on the road to independent thinking and a constructivist approach to problem solving. Rather than depending on memorized procedures, the student will actually think through the problem and develop a rationale for solution.

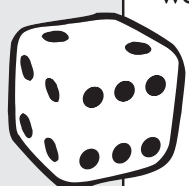
Fluency with number and operation is a critical skill for the journey into more complex mathematical challenges. Like basal texts, the activities in this book (and series) provide students with ample opportunity to practice basic skills and facts. The difference is that the dice activities are highly engaging, employing game theory as the motivating force to keep students focused.

The preceding book in the series stressed multiplication facts; *Dice Activities for Mathematical Thinking* emphasizes more sophisticated mathematical concepts such as prime and composite numbers, factorials, summation, square roots, and square numbers. These mathematical concepts are key elements of many algebraic patterns, and square numbers appear on a frequent basis. Students therefore need a working knowledge and quick recognition of these numbers. The activities in this book provide that knowledge base, helping students develop a fluency in working with a level of number that is more mathematical than arithmetic.

Factorials and summations surface in many of the problems that abound in assessment and instructional programs. A facility in working with these numbers and the ability to use them to strategize and to solve problems is invaluable. Students can function at various levels of problem solving in approaching factorial and summation-related challenges. Consider the problem of summation 4, noted as $\sum 4$. When asked to solve $\sum 4$, students should expand it to read: $\sum 4 = 4 + 3 + 2 + 1 = 10$. Depending on their mathematical competency, a variety of strategies will be employed. Some students will just begin to add the numbers as in the following: $4 + 3 = 7$, $7 + 2 = 9$, and $9 + 1 = 10$.

Other students might consider that they just recently found the solution to $\sum 3$ (= 6) and since $\sum 4$ includes $\sum 3 + 4$, then the answer merely requires them to add $6 + 4$, which equals 10. Some students will look for patterns to add sequences of numbers and strategize that the summation of even numbers might be an algebraic pattern of $[.5n] \times n + [.5n]$ … in this case, half of 4 = 2, times 4 = 8, plus half of 4 = 10. The critical issue is that whatever strategy a student employs makes sense to that student and is not merely a repetition of the teacher-taught strategy. Students are more apt to remember their own solutions or at least be able to reconstruct them. Herein lies the power of differentiated instruction. Students need to be allowed to employ their own strategies to solve problems. This constructivist approach is the foundation of the major mathematical programs used in this country and the underpinnings of the state assessment instruments—the latter witnessed by the aspect of the test that requires students to explain their thinking and strategies.

This is not to imply that there are not multitudes of ways a teacher can differentiate learning within a lesson. Take, for example, an activity with prime numbers. For more mathematically competent students, a teacher may have students use a limited number of dice, say 4, to make equations that equal as many prime numbers as possible. For students who have not yet developed a fluency with number or mathematical thinking, the same problem can be posed in the same lesson while using 6 dice instead. And even more powerful, rather than the teacher assigning the use of 4 dice to some and 6 dice to others, the option can be left to the students. Students can operate at their own level of success and gradually move to more challenging possibilities as their level of confidence increases.



It should be noted that most of the activities in this book are designed for two teams of two students competing against each other. The team approach allows students to check their thinking and share strategies with their partner. In cases, however, where the more assertive partner does all the thinking and the less assertive partner willingly lets him/her take the lead, it is better to have one student play against another, since it requires both students to think, strategize, and make decisions. Having students play one-on-one is also a good assessment tool, as the teacher can more closely monitor individual progress. Whether playing individually or as a member of a team, students benefit from working with a shoulder partner when introduced to these activities.

Many of these activities can be explored on a larger scale. Take, for instance, the classic problem of using four 4s to write the numbers 1 to 100. This could be done as a whole-class activity, challenging the students to collaborate on solutions. A dramatic challenge would be for several classrooms at the same grade level to compete with one another. As the students closed in on all the solutions, they would discover that certain numbers are much more difficult and would employ the use of factorials, squaring numbers, and/or summation. Initially it might seem impossible to square any of the numbers, since only 4s can be used. Inserting a 4 into a square root symbol produces the number 2, which allows students to square numbers.

The joy of mathematical thinking is an experience that all children deserve, regardless of their mathematical ability. In a nutshell, that should be the kernel of the No Child Left Behind legislation. All children deserve and need a mathematical education that allows them to think and not merely memorize. But at the same time, it is folly to assume that all children have the same intellectual capacity and to legislate on the basis of that assumption. The educators of this country need to stand firm as to what our children deserve and what it means to be truly educated mathematically.

—Mary Saltus and Chet Delani

Meeting the NCTM Standards

NCTM STANDARDS Grades 5–8	*Dice Graph Activities Pages 1–13*	*Table Completion Chart Activities Pages 15–24*	*Four in a Row Activities Pages 25–36*	*Square Off Activities Pages 37–48*	*Cross Over Activities Pages 49–60*	*Tic-Tac-Toe Activities Pages 61–83*	*Aim For Activities Pages 85–91*	*Independent Explorations Pages 93–118*
Number and Operations								
Place value								
Equivalent representations	✗	✗	✗	✗	✗	✗	✗	✗
Positive and negative integers	✗	✗	✗	✗	✗	✗	✗	✗
Fractions, decimals, and percents								
Ratios and proportions								
Exponential notation	✗	✗	✗	✗	✗	✗	✗	✗
Factors, multiples, prime factorization	✗	✗	✗	✗	✗	✗	✗	✗
Compare quantities using integers							✗	
Relationships between operations	✗	✗	✗	✗	✗	✗	✗	✗
Properties of operations	✗	✗	✗	✗	✗	✗	✗	✗
Squaring and square roots	✗	✗	✗	✗	✗	✗	✗	✗
Fluency with operations	✗	✗	✗	✗	✗	✗	✗	✗
Select appropriate methods	✗	✗	✗	✗	✗	✗	✗	✗
Develop and analyze algorithms							✗	✗
Estimation	✗	✗	✗	✗	✗	✗	✗	✗
Algebra								
Generalize numeric patterns		✗						✗
Use algebraic symbols	✗	✗	✗	✗	✗	✗	✗	✗
Model situations with equations	✗	✗						✗
Data Analysis and Probability								
Represent and analyze data	✗	✗						
Predict outcomes	✗	✗	✗	✗	✗	✗		
Problem Solving	✗	✗	✗	✗	✗	✗	✗	✗
Reasoning and Proof	✗	✗	✗	✗	✗	✗	✗	✗
Communication		✗	✗	✗	✗	✗	✗	✗

Directions for Four in a Row Activities

Objectives

- Develop a working knowledge of the mathematical concepts of:
  - Square numbers
  - Square roots
  - Prime numbers
  - Factorials
  - Summations
  - Positive and negative integers
- Develop an awareness of an opponent's possible moves.
- Analyze an opponent's possible moves in order to develop a blocking strategy.
- Identify the role of luck versus skill in an activity using dice.
- Develop communication and cooperation skills by working in teams of two students.

Introduce the **Four in a Row** activities by demonstrating on an overhead and playing against the class. Two teams with two students on a team are suggested. Teams give students an opportunity to discuss moves and strategies and provide a check on correct computation.

How to Play

- Teams toss die or dice, depending on the activity, and perform the required computation—operations differ for each activity. For example:

  In the square root activities, if the sum of the two dice tossed is 7, then players look for the number for which 7 is the square root.
  In the factorial activities, if the sum of the two dice tossed is 5, then players look for 5!, or 120 (5! = 5 x 4 x 3 x 2 x 1 = 120).
- Teams attempt to line up four tokens vertically, horizontally, or diagonally before the opposing team does.
- The first team to align four tokens in a row wins.

Suggestions

- Before placing a token on the chart, team members explain how they arrived at a solution.
- If students are struggling with determining the closest square number or prime number, suggest that they refer to the Square Number Chart and Prime Number Chart on pages viii–ix.

Discussion

- This activity is similar to the games *Othello* and *Pente*, where defense is important. How does the toss of the dice influence strategy? Is this activity more a game of defense or offense?
- Does this activity involve more luck or skill?
- Keep a recording of each dice toss. Which combinations were tossed the most? The least?

Four in a Row
Square the Die Chart

- Each team tosses a die.
- Higher number goes first.
- Each team chooses a color token.

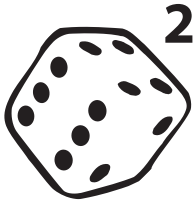
How to Play

1. Toss a die. Square the number (multiply the number by itself—for example, 7 × 7).

2. Place a token on the square number.

3. The first team to get 4 tokens in a row, vertically, horizontally, or diagonally, wins.

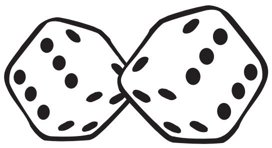
2



36	1	16	25	9	4	16
25	9	36	1	4	16	1
1	16	4	9	25	26	4
4	25	9	16	36	1	25
9	36	1	4	16	25	9
16	4	25	36	1	9	36
25	9	16	1	36	4	25

Four in a Row
Square Two Dice Chart

- Each team tosses a die.
- Higher number goes first.
- Each team chooses a color token.



How to Play

1. *Toss 2 dice. Find the sum. Square the sum (multiply the sum by itself—for example, 7 × 7).*
2. *Place a token on the square number.*
3. *The first team to get 4 tokens in a row, vertically, horizontally, or diagonally, wins.*

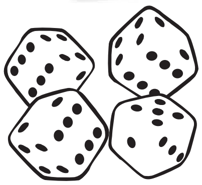
36	121	9	25	81	16	4
49	64	144	100	36	25	81
25	49	36	64	4	81	64
64	36	81	49	121	16	49
100	25	4	16	36	49	144
16	81	121	49	100	9	64
9	36	64	144	49	25	100

- Each team tosses a die.
- Higher number goes first.
- Each team chooses a color token.

Four in a Row

Closest Square Number Chart

How to Play



1. Toss 2 red dice and find the sum. Toss 2 green dice and find the sum.

2. Multiply the sum of the red dice by the sum of the green dice.

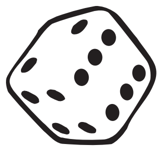
3. Place a token on the closest square number to the product. Example: red dice = 5 + 2 = 7, green dice = 4 + 4 = 8, 8 × 7 = 56; 56 is near both 49 and 64 but closer to 49; 49 is the closest square number.

4. First team to get 4 tokens in a row, vertically, horizontally, or diagonally, wins.

36	121	9	25	81	16	4
49	64	144	100	36	25	81
25	49	36	64	4	81	64
64	36	81	49	121	16	49
100	25	4	16	36	49	144
16	81	121	49	100	9	64
9	36	64	144	49	25	100

Four in a Row
One-Die Square Root Chart

- Each team tosses a die.
- Higher number goes first.
- Each team chooses a color token.



How to Play

1. *Toss a die. The number tossed is the solution to which square root expression on the chart?*
2. *Place a token on that expression.*
3. *The first team to get 4 tokens in a row, vertically, horizontally, or diagonally, wins.*

$\sqrt{36}$	$\sqrt{1}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{9}$	$\sqrt{4}$	$\sqrt{16}$
$\sqrt{25}$	$\sqrt{9}$	$\sqrt{36}$	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{16}$	$\sqrt{1}$
$\sqrt{1}$	$\sqrt{16}$	$\sqrt{4}$	$\sqrt{9}$	$\sqrt{25}$	$\sqrt{26}$	$\sqrt{4}$
$\sqrt{4}$	$\sqrt{25}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{36}$	$\sqrt{1}$	$\sqrt{25}$
$\sqrt{9}$	$\sqrt{36}$	$\sqrt{1}$	$\sqrt{4}$	$\sqrt{16}$	$\sqrt{25}$	$\sqrt{9}$
$\sqrt{16}$	$\sqrt{4}$	$\sqrt{25}$	$\sqrt{36}$	$\sqrt{1}$	$\sqrt{9}$	$\sqrt{36}$
$\sqrt{25}$	$\sqrt{9}$	$\sqrt{16}$	$\sqrt{1}$	$\sqrt{36}$	$\sqrt{4}$	$\sqrt{25}$