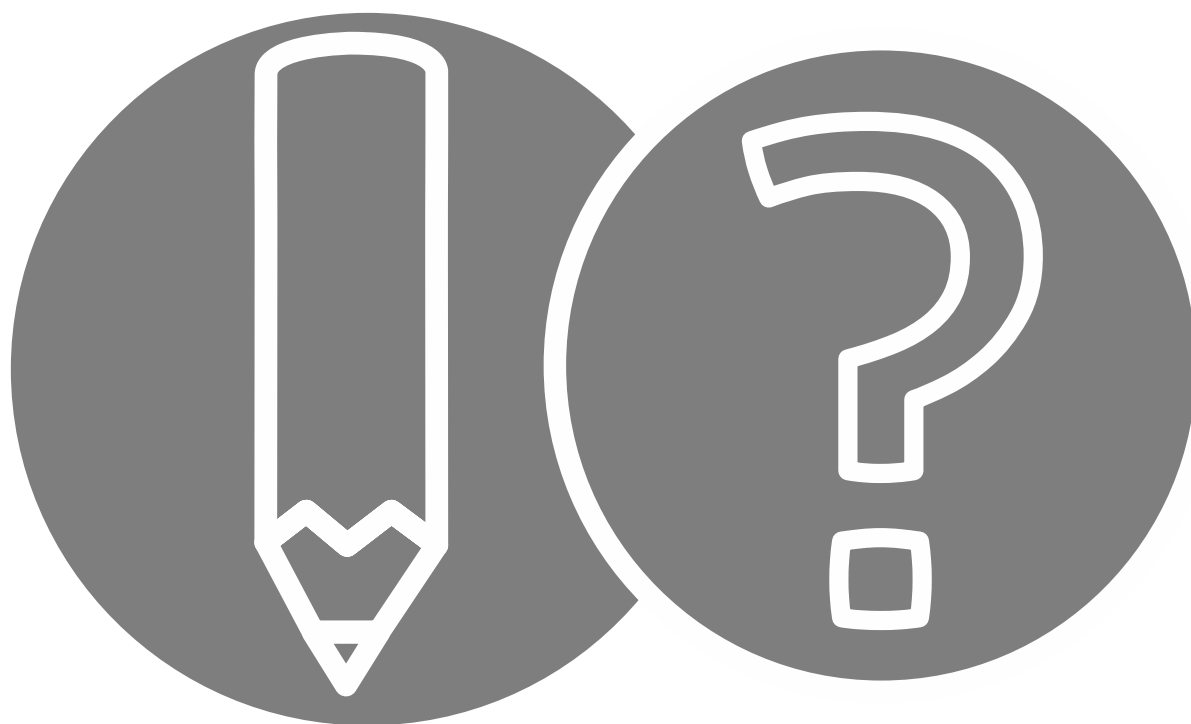# Draw & Solve
Word Problems

Rob Madell and Laura Zoe Dombrowski

Modeling Word Problems for
Multiplication and Division

Contents

Introduction

The ability to model word problems is the basis of all of whole number arithmetic. The Math Standards define 17 different types of word problems that children are expected to be able to model by the end of Grade 3. Of those 17, children are expected to have mastered 11—those involving addition and subtraction—by the end of first grade. The expectation is that children will master the six types involving multiplication and division by the end of third grade. Here is an example of one of those types.

Equal Groups/Unknown Product

> 3 girls were having lunch at school. Each girl had 4 carrots in her lunchbox. How many carrots did the girls have altogether?

We might teach children to model this problem with counters:

1. Show me the 3 counters that will be the girls' lunchboxes.

2. For each lunchbox, show me the 4 counters that will be the carrots in that lunchbox.

3. Count how many carrots there are altogether.

We might also teach children to model this problem with paper and pencil:

1. Draw 3 small squares that will be the girls' lunchboxes.

2. For each lunchbox, draw 4 tally marks that will be the carrots in that lunchbox.

3. Count how many carrots there are altogether.

This book, together with the others in the series, is intended to help children model the Math Standards problem types with paper and pencil. The six multiplication and division types that are required by the end of Grade 3 are included here in this volume.

How to Use This Book

Start by reminding yourself of the six multiplication and division problem types that children are expected to learn to model by the end of third grade. The Introduction to Multiplication discusses the two types of multiplication problems and the Introduction to Division discusses the four types of division problems.

Next, introduce your students to solving new types of word problems with physical models. Use counters or other manipulatives for demonstrations and discussions with the whole group, with small groups, and ideally with individual children.

At some later point you can introduce paper-and-pencil models and then have children work largely on their own with the worksheets provided here. Paper-and-pencil models have at least one advantage over physical models. At the end of class you will be able to collect and review each student's work.

We have used worksheets like these with our students and are happy with the results. We hope that you have a similar experience.

A Note About Names

In writing here about the various problem types, we have used the names assigned to those types by the Math Standards. But the only names that we use with our students are *addition*, *subtraction*, *multiplication*, and *division*. We do sometimes make the distinction, for example, between "one type of multiplication problem" and "another type of multiplication problem."

A Note About Modeling

Eventually children will learn to represent word problems with equations, and to solve those word problems, and to solve those equations, with strategies beyond counting. For example, the problem about carrots can be represented by the equation

$$3 \times 4 = \boxed{}$$

and can be solved by counting by fours—4, 8, 12. But first children must learn what these word problems mean—in this case that the carrots should be put together and counted one by one. These books are about teaching children what word problems mean.

A Note About Drawing

Some children like to make elaborate drawings. They may want to draw the girls and the lunchboxes and the carrots. They may want to give the girls faces and hair. They may want to put pictures of superheroes on the lunchboxes. But none of this detail is essential to the mathematics. In fact, the Standards of Mathematical Practice encourage teachers to help children to "*decontextualize*—to abstract a given situation and represent it symbolically . . ." These books support that standard. We think that you should encourage children to make very simple drawings.

A Note About Understanding vs. Memorizing

Some children come to school understanding some types of word problems. **Add To/Result Unknown** is a good example. Other types of word problems can be difficult for children. **Take From/ Start Unknown** and **Compare/Bigger Unknown** are good examples. To help them we advocate direct instruction in modeling. But to be clear, we are not arguing for mechanical, rote learning. We merely believe in putting difficult problems in front of children and helping them to understand those problems. We don't advocate having them memorize steps that they do not understand.

—Rob Madell and Laura Dombrowski

It is also important that children try to write their own word problems. This girl confuses 4 bunches (or baskets) of cherries with 4 cherries.

Introduction to Multiplication

The Math Standards define two different types of multiplication word problems that children are expected to be able to model by the end of Grade 3. The Standards assume that both of these problem types will have been introduced in Grade 2.

Equal Groups/Unknown Product

> A bakery has 4 boxes of cookies in its window. Each box has 3 cookies in it. How many cookies are there altogether?

Here are the steps in modeling this problem.

1. Draw 4 squares to represent the 4 boxes.

2. In each square draw 3 circles to represent the cookies.

3. Count the cookies.

Teaching Tip 1 – A More Difficult Example

Here is a somewhat more difficult Equal Groups/Unknown Product problem.

A bakery has 4 kinds of pie—apple, cherry, blueberry, and rhubarb. The bakery has 3 of each kind of pie in the window. How many pies are in the window?

With problems like this one, you may want to have students start by somehow representing the different categories. In this case, the kinds of pie might be represented by the letters A, C, B, and R for apple, cherry, blueberry, and rhubarb.

Teaching Tip 2 – What Does 4 × 3 Mean?

There are different practices with respect to whether 4 × 3 represents 4 groups of 3 or 3 groups of 4. Of course, even though it is not obvious, it does turn out that, with either meaning, 4 × 3 = 12. For that reason many teachers use 4 × 3 to represent both 4 groups of 3 and 3 groups of 4. We think that a better practice would be to decide, for your classroom, what you mean by 4 × 3, and more generally, what you mean by $a \times b$.

Here is an example of a multiplication problem involving an array. Remember:

- An array is a collection of objects arranged in rows and columns.

- Each row must have the same number of objects in it.

- Each column must have the same number of objects in it.

- An array with, for example, 4 rows and 3 columns is called a "4-by-3" array.

Arrays/Unknown Product

> A bakery has a display of cookies in their window. The cookies are arranged in an array with 4 rows and 3 columns. How many cookies are in the display?

Assuming, of course, that children understand what an array is, the steps in modeling an Array problem are simple. Here are the steps for the Array problem above.

1. Draw the 4-by-3 array of cookies.

2. Count the cookies in the array.

These two examples should make it clear that Equal Groups problems and Array problems are really not all that different from one another. In fact, the real reason for introducing arrays is that it is very useful to have children use them to model **Equal Groups/Unknown Product** problems.

A box of cookies can be represented as a row in an array. Four boxes of cookies with 3 cookies in each box can be represented by an array with 4 rows and 3 columns.
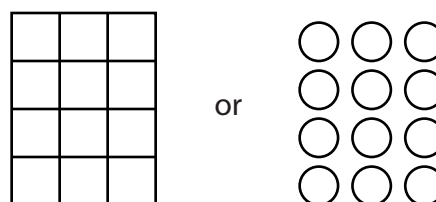
When Equal Group problems are represented by arrays, it is easy to see, for example, that there are the same number of cookies in 4 boxes (rows) of 3 as in 3 boxes (columns) of 4 (**the Commutative Property**). And it is easy to see, for example, that the number of cookies in 3 boxes of 14 is the same as in 3 boxes of 10 together with 3 boxes of 4 (**the Distributive Property**).
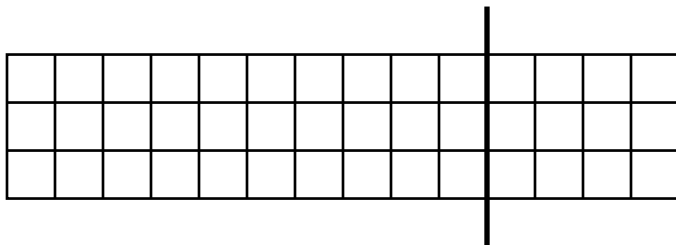
Seeing arrays in the context of solving Equal Groups problems can help children develop strategies beyond one-by-one counting.

Equal Groups/Unknown Product problems begin on page 3.

Arrays/Unknown Product problems begin on page 13. (On these pages, only the first problem is actually about an array. The second problem describes a situation involving equal groups that students are asked to represent using an array.)

Name _____ Date _____

1. A girl put her toy trucks into an array on the floor. She made 3 rows and 4 columns. How many trucks are in her array?
   _12_

   Draw an array.

   Write an equation. _3 × 4 = 12_

2. A parking garage has 4 floors. There are 5 cars parked on each floor. How many cars are in the garage? _____

   Draw an array.

   Write an equation. _5 × 4 = 20_

3. 6 × 3 = _18_

   Draw an array.

Notice that for this boy, 3 × 4 means 4 bunches of 3, and 5 × 4 means 4 bunches of 5. If he were consistent, then 6 × 3 would mean 3 bunches of 6.

Name _____ Date _____

1. A town has 3 schools. There are 7 teachers in each school. How many teachers are there altogether? _____

Draw a picture.

Write an equation. _____

2. 6 × 5 = _____

Draw a picture.

3. Write a word problem for 4 × 3 = _____.

_____

_____

_____

_____

Name _____ Date _____

1. A girl put her toy trucks into an array on the floor. She made 3 rows and 4 columns. How many trucks are in her array? _____

Draw an array.

Write an equation. _____

2. A parking garage has 4 floors. There are 5 cars parked on each floor. How many cars are in the garage? _____

Draw an array.

Write an equation. _____

3. $6 \times 3 =$ _____

Draw an array.

Introduction to Division

The Math Standards define four different types of division word problems that children are expected to be able to model by the end of third grade.

Equal Groups/Group Size Unknown

> 3 children want to share 18 apples so that each child gets the same number of apples. How many apples should each child get?

Here is how to model this problem.

1. Draw the 3 children.
2. Distribute the 18 apples among the 3 children.
3. Count how many apples each child got.

> ### Teaching Tip 1 – A More Difficult Example
>
> Here is a somewhat more difficult Equal Groups/Group Size Unknown problem.
>
> *A baker baked 3 kinds of pies—apple, blueberry, and cherry. There are 18 pies altogether, with the same number of each kind. How many pies of each kind are there?*
>
> With problems like this one, you may want to have students start by somehow representing the different categories—in this case they might use the letters A, B, and C for apple, blueberry, and cherry. They can then distribute the 18 pies under those 3 headings.

Equal Groups/Number of Groups Unknown

> A grocer has 18 apples. She wants to put the apples into bags, with 3 apples in each bag. How many bags does the grocer need?

Here is how to model it.

1. Draw the 18 apples.
2. Circle groups of 3 apples.
3. Count how many groups of 3 there are.

You should notice that these two Equal Groups problems really are different from one another. They are not modeled in the same way. On the surface, there is no reason to expect them to have the same answer.

- If there are 18 apples and if 3 children share them, then a detailed model includes 21 objects altogether.

- If there are 18 apples in bags of 3, then there are 6 bags and 24 objects altogether.

A child might understand (i.e., be able to model) one of these problems but not the other.

> ### Teaching Tip 2 – Group Size Unknown vs. Number of Groups Unknown
>
> It is important for children to have experiences with both Group Size Unknown problems and Number of Groups Unknown problems and to get practice in modeling each in an appropriate way. You will need to be especially careful when helping children. For example, if a child is struggling with 3 children sharing 18 apples, it may not be helpful to suggest that he count by threes to 18. It is true that the number of threes in 18 will give him the right answer, but it is not at all obvious that it should.

The two remaining types of division problems involve arrays. But these Array problems are not very different from the corresponding Equal Groups problems.

Arrays/Group Size Unknown

> 18 apples are arranged in an array with 3 rows. How many apples are in each row?

Assuming that children know what arrays are, and can draw them, there is little difference between putting 18 apples into 3 rows, and sharing 18 apples among 3 children. In fact, a good way to model 3 children sharing 18 apples is with an array with 3 rows. Each of the 3 children gets the 6 apples in one of the 3 rows.
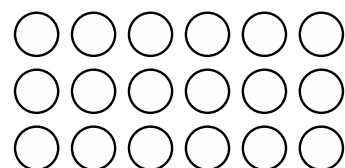
One of the advantages of this representation is that it helps children to see that the number of apples each child gets (there are 6 apples in each of the 3 rows) is the same as the number of threes in 18 (the number of columns).

This representation also helps children to see the connection between sharing and multiplication. In other words, using arrays in the context of solving Equal Groups problems should help children develop strategies beyond one-by-one counting. This is the main reason for introducing word problems involving arrays.

Arrays/Number of Groups Unknown

18 apples are arranged in an array with 3 apples in each column. How many columns are in the array?

Assuming again that children know what arrays are, and can draw them, there is little difference between putting 18 apples into an array with columns of 3, and putting 18 apples into bags of 3. In fact, a good way to model putting 18 apples into bags of 3 is with an array with columns of 3. The number of bags of apples is the number of columns.

As above, there are advantages to having children use arrays in their modeling of Equal Groups

problems. Those arrays should help them to develop strategies beyond one-by-one counting.

One final comment: It is worth noticing that when 18 apples are divided into 3 equal groups, the number of apples in each group is the same as the number of threes in 18, and so the equation $18 \div 3 = \square$ can represent both problems. Children can think about that equation in either way. There may be four different types of division problems, but regardless of how $18 \div 3$ is interpreted, $18 \div 3 = 6$.

Equal Groups/Group Size Unknown problems begin on page 25.

Equal Groups/Number of Groups Unknown problems begin on page 35.

Arrays/Group Size Unknown problems begin on page 45.

Arrays/Number of Groups Unknown problems begin on page 55.

(On each of the pages devoted to Array problems, only the first problem is actually about an array. The second problem describes a situation involving equal groups that students are asked to represent using an array.)

Notice that this girl interpreted 30 ÷ 5 in two different ways—first, as "how many 5s are in 30," and then as "30 divided into 5 equal groups."

Name _____ Date _____

1. A park has 4 ponds. There are the same number of ducks in each pond and there are 24 ducks altogether. How many ducks are in each pond? _____

Draw a picture.

Write an equation. _____

2. 25 ÷ 5 = _____

Draw a picture.

3. Write a word problem for 28 ÷ 7 = _____.

_____

_____

_____

_____

Name _____ Date _____

1. A bookshelf holds 42 books. Each shelf has 6 books on it. How many shelves are there? _____

Draw a picture.

Write an equation. _____

2. $9 \div 3 =$ _____

Draw a picture.

3. Write a word problem for $40 \div 5 =$ _____.

_____

_____

_____

_____

_____