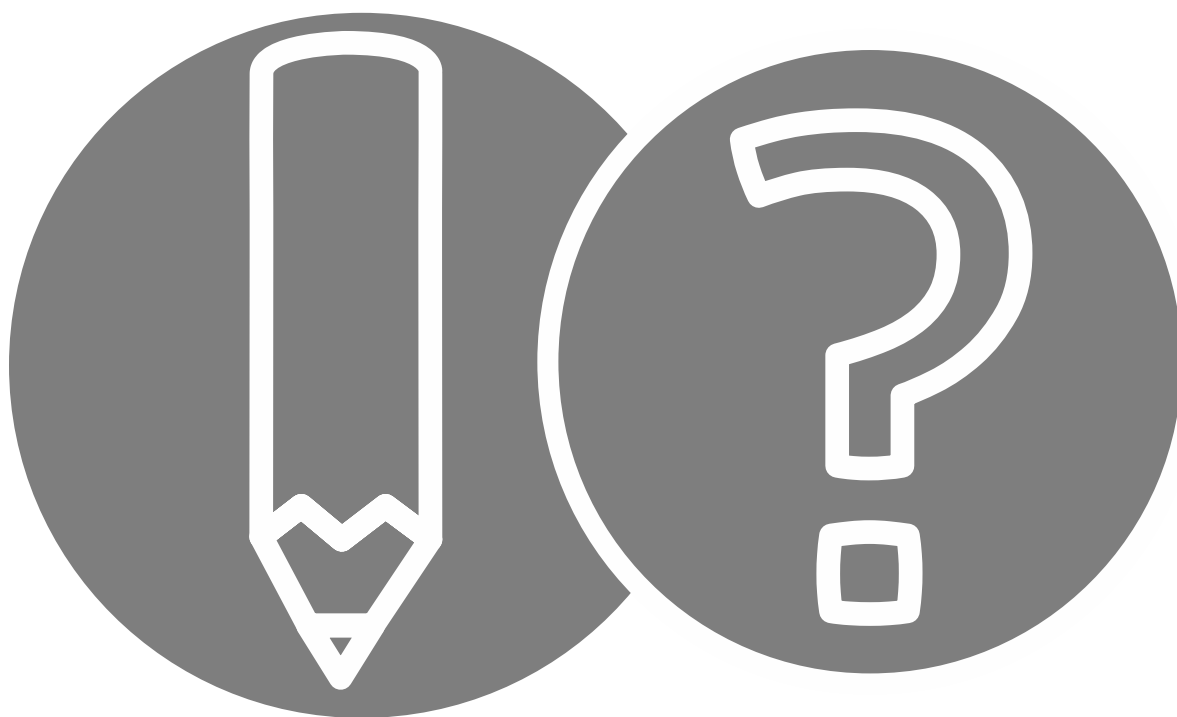


Grade
1

Draw & Solve

Word Problems



Rob Madell and Laura Zoe Dombrowski

Modeling Word Problems for
Addition and Subtraction
within 20





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Introduction

The ability to model word problems is the basis of all of whole-number arithmetic. The Math Standards define 11 different types of word problems that children are expected to be able to model by the end of Grade 1. Here is an example of one of those types.

Put Together/Total Unknown

9 apples are in a tree.

7 apples are on the grass.

How many apples are there in all?

We might teach children to model this problem with counters:

1. Show me the 9 counters that will be the apples in the tree.
2. Show me the 7 counters that will be the apples on the grass.
3. Put the apples together and count how many there are altogether.

We might also teach children to model this problem with paper and pencil:

1. Draw 9 small circles that will be the apples in the tree.
2. Draw 7 small circles that will be the apples on the grass.
3. Count how many apples there are altogether.

This book, together with the others in the series, is intended to help children model the Math Standards problem types with paper and pencil. The 11 types that are required by the end of Grade 1 are included here in this volume.

How to Use This Book

Start by reminding yourself of the 11 different problem types that children are expected to learn to model by the end of Grade 1.

The Introduction to Addition discusses the four types of addition problems, and the Introduction to Subtraction discusses the seven types of subtraction problems.


Next, introduce your students to solving new types of word problems with physical models. Use counters or other manipulatives for demonstrations and discussions with the whole group, with small groups, and ideally with individual children.

At some later point you can introduce paper-and-pencil models and then have children work largely on their own with the worksheets provided here. Paper-and-pencil models have at least one advantage over physical models. At the end of class you will be able to collect and review each student's work.

Take From/Change Unknown - 3

Name _____

There are 9 seeds.
A bird eats some.
There are 6 seeds left.
How many seeds did the bird eat? 3



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First-grade students easily learn to make simple drawings. Circles can represent chickens. The letter P can represent a pig. Here seeds are represented very simply—by small dots.

We have used worksheets like these with our students and are happy with the results. We hope that you have a similar experience.

A Note About Names

In writing here about the various problem types, we have used the names assigned to those types by the Math Standards. But the only names that we use with our students are *addition*, *subtraction*, *multiplication*, and *division*. We do sometimes make the distinction, for example, between “one type of addition problem” and “another type of addition problem.”

A Note About Modeling

Eventually children will learn to represent word problems with equations, and to solve those word problems, and to solve those equations, with strategies beyond counting. For example, the problem about apples can be represented by the equation

$$9 + 7 = \square$$

and can be solved by reasoning that since $7 + 7 = 14$, then $9 + 7$ must equal 16. But first children must learn what these word problems mean—in this case that the apples should be put together and counted one by one. These books are about teaching children what word problems mean.

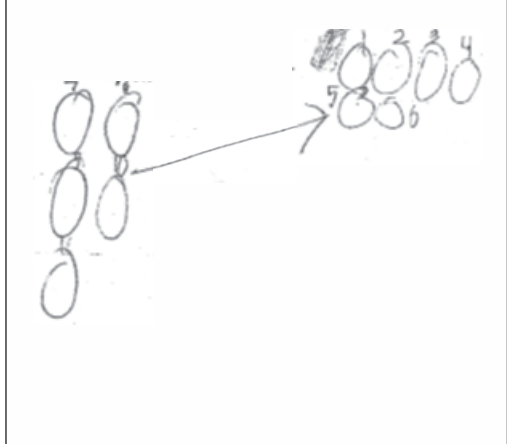
A Note About Drawing

Some children like to make elaborate drawings. For the problem about apples, they may want to draw apples with stems, trees with leaves, and flowers growing in the grass. They may even want to draw the sun and clouds. But none of this detail is essential to the mathematics. In fact, the Standards of Mathematical Practice encourage teachers to help children to “*decontextualize*—to abstract a given situation and represent it symbolically . . .” These books support that standard. We think that you should encourage children to make very simple drawings.

64 Add To/Change Unknown - 4

Name _____

6 chickens are eating.
Then some more chickens come to eat.
Now there are 11 chickens eating.
How many chickens came to eat? 5



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An advantage of drawings over manipulatives is that they leave a record of how the child solved the problem. It is clear that this child counted out 6 “chickens,” then counted out more until he got to 11, then counted out how many he had added to the original 6.

A Note About Understanding vs. Memorizing

Some children come to school understanding some types of word problems. **Add To/Result Unknown** is a good example. Other types of word problems can be difficult for children. **Take From/Start Unknown** and **Compare/Bigger Unknown** are good examples. To help them we advocate direct instruction in modeling. But to be clear, we are not arguing for mechanical, rote learning. We merely believe in putting difficult problems in front of children and helping them to understand those problems. We don't advocate having them memorize steps that they do not understand.

A Note About Reading

We have tried to make the word problems in this book as easy to read as possible. We hope that this allows for the challenge of the word problems to be mathematical rather than one of reading. A complete list of all the words used can be found in the word list on page 95.

But in your class discussion we suggest that you restate the problems to sound more natural. For example, where a problem reads,

2 ducks sit in the grass.

3 more ducks come to sit in the grass.

How many ducks are in the grass now?

you might say instead,

2 ducks were sitting in the grass.

Then 3 more ducks came to sit with them.

How many ducks are in the grass now?

—Rob Madell and Laura Dombrowski

Introduction to Addition

The Math Standards define 11 different types of word problems that children are expected to be able to model by the end of Grade 1. Of these 11, four involve addition.

Children need to be helped to see that in spite of important differences among these four types, each can be understood as the joining of two sets of objects, and each can be solved by counting the objects in the joined sets altogether. That is why the same strategies and algorithms can be used to solve all four types of problems.

Add To/Result Unknown

A boy had 4 dolls.

Then his sister gave him 7 more dolls.

How many dolls does the boy have now?

Put Together/Total Unknown

4 pigs are in the mud.

7 pigs are on the grass.

How many pigs are there altogether?

These two types of word problems are typically introduced in Kindergarten—usually with sums less than 10. Most children have little difficulty learning to model them. The steps in making those models are identical.

Step	Add To/ Result Unknown	Put Together/ Total Unknown
1	Draw the 4 dolls the boy had.	Draw the 4 pigs in the mud.
2	Draw the 7 dolls his sister gave to him.	Draw the 7 pigs on the grass.
3	Count the dolls altogether.	Count the pigs altogether.

In the pages that follow, you will find examples with sums as large as 20. **Add To/Result Unknown** problems begin on page 3 and are followed by **Put Together/Total Unknown** problems, beginning on page 11.

Take From/Start Unknown

Some cats were sitting on a bed.

4 of the cats jumped off the bed onto the floor.

Then 7 cats were still on the bed.

How many cats were on the bed at first?

Help your students to visualize a “mental movie” of the scene that this problem describes. At the end of that movie there are 7 cats sitting on a bed and there are an additional 4 cats on the floor. Then help them to roll that movie backwards. At the start of the movie the 4 cats on the floor are on the bed. Before those 4 jumped off the bed, there were $7 + 4 = 11$ cats on the bed.

So this really is an addition problem! A hard addition problem! It is hard because it is hard to play that movie backwards. It is even hard to understand that you *should* play the movie backwards.

We can help children to imagine the movie by helping them to model the problem following these steps:

1. Draw the 7 cats that are still on the bed.
2. Draw the 4 cats that jumped off the bed.
3. Count the cats that were on the bed at first.

Take From/Start Unknown problems begin on page 27.

Teaching Tip

Play the Movie Forward – Some children will try to solve this problem by playing the movie forward. You may even want to encourage that. At the start of the movie there are some cats on the bed—but we don’t know how many. Some children may say, “What take away 4 equals 7?” Other children may even write the equation $\square - 4 = 7$. To help these children solve the problem, have them guess at what that missing number is—and have them check to see if they are right. Doing that is a good introduction to algebra and may help children to play the movie backward. But, playing it backward is what is important if children are to see the problem as an example of addition. Only when the movie is played backward will they see that the two sets of cats can be joined together and counted to solve the problem.

Compare/Bigger Unknown

A little dog ate 4 dog treats.

A big dog ate 7 more treats than the little dog.

How many treats did the big dog eat?

This type of problem also turns out to be difficult for children to understand. Many of them interpret “a big dog ate 7 more treats than the little dog” to mean that the big dog ate *exactly* 7 treats. Somehow they don’t understand what those words really mean.

There is a big difference between modeling Compare/Bigger Unknown problems and modeling all the other addition problems. Whether acted out with physical objects or represented with paper and pencil, there must be 15 dog treats in any proper representation of this problem—not 11, as in the models above. (But it is still an addition problem because the treats that the big dog ate are the 4 that match what the little dog ate, together with 7 more.)

Here are the steps of this model.

1. Draw 4 treats for the little dog.
2. Draw 4 treats for the big dog so that she has the same number of treats as the little dog.
3. Draw 7 more treats for the big dog.
4. Count how many treats the big dog has.


Compare/Bigger Unknown problems begin on page 27.

This is a good example of the results of direct instruction in modeling. We taught the children to use P’s and C’s to represent pigs and cows. And we taught them to start by showing the same number of pigs as cows.

Take From/Start Unknown - 3

Name _____

Some cows are in the barn.
3 of them go out.
There are 7 cows left in the barn.
How many cows were in the barn at first? 10




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A very simple representation of 7 cows in a barn and 3 cows outside the barn—the scene at the end of the “movie.” All the essential information has been captured.

Compare/Bigger Unknown - 1

Name _____

6 pigs are on a farm.
There are 2 more cows than pigs.
How many cows are on the farm? 8



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Name _____

6 birds sit in a tree.

3 more birds fly there.

How many birds are in the tree now? _____

Name _____

3 bears eat.

6 bears sleep.

How many bears are there in all? _____

Name _____

Some toys are on the floor.

A boy puts 4 of them on a table.

There are 2 toys left on the floor.

How many toys were on the floor at first? _____

Name _____

4 pigs are on a farm.

There are 4 more cows than pigs.

How many cows are on the farm? _____

Introduction to Subtraction

The Math Standards define 11 different types of word problems that children are expected to be able to model by the end of Grade 1. Of those 11, seven involve subtraction.

Children need to be helped to see that in spite of important differences among these seven, each can be understood as removing some of the objects from a set, and each can be solved by counting the objects that remain. That is why the same strategies and algorithms can be used to solve all seven types of problems.

Take From/Result Unknown

13 apples were on the table.

I ate 5 of the apples.

How many apples are on the table now?

Take Apart/Addend Unknown

13 apples are on the table.

5 of the apples are red and the rest are green.

How many apples are green?

These two types of word problems are typically introduced in Kindergarten—usually with numbers less than 10. Most children have little difficulty learning to model them. The steps in making those models are identical.

Step	Take From/ Result Unknown	Take Apart/ Addend Unknown
1	Draw the 13 apples.	Draw the 13 apples.
2	Show which 5 apples were eaten.	Show the 5 apples that are red.
3	Count the apples that are left.	Count the apples that are green.

Take From/Result Unknown problems happen over time and explicitly describe the separating of a set into two parts. **At the start**, 13 apples were on the table. **Then** I ate 5 of the apples.

Take Apart/Addend Unknown problems describe a set (**the 13 apples**) that has two parts (**the red apples and the green apples**). Nothing happens over time, and nothing is said about these two parts physically separating from one another.

In spite of these distinctions, the chart above shows that there is little or no difference in the way these problems are modeled.

In the pages that follow you will find examples with numbers as large as 20. **Take From/Result Unknown** problems begin on page 39 and are followed by **Take Apart/Addend Unknown** problems, beginning on page 47.

Take From/Change Unknown

13 apples were on the table.

I ate some of the apples.

Then there were 5 apples left.

How many apples did I eat?

As with the problems above, most children will have little trouble with the first step in making the model. They should draw the 13 apples. But some children may need help in understanding that the set of 13 apples consists of some that were eaten and some that are still left on the table. To find out how many were eaten, they should first remove the ones that are left.

Here are the steps of this model.

1. Draw the 13 apples.
2. Show which 5 apples are left.
3. Count the apples that were eaten.

Take From/Change Unknown problems begin on page 55.

Teaching Tip 1

Introduction to Algebra – After drawing the 13 apples, some children will want to guess at how many apples were eaten. They may even write the equation $13 - \square = 5$. That is a good introduction to algebra and should be encouraged. But it is more important for first-graders that they see this problem as an example of a subtraction problem—and that requires them to see that to find the apples that were eaten they can take away the apples that are left.

Add To/Change Unknown

5 brown squirrels were in a tree.

Then some black squirrels ran up the tree.

Now there are 13 squirrels in the tree.

How many black squirrels ran up the tree?

Add To/Start Unknown

Some brown squirrels were in a tree.

Then 5 black squirrels ran up the tree.

Now there are 13 squirrels in the tree.

How many brown squirrels are in the tree?

Help your students to imagine two short movies picturing the events in these two problems. Both movies start with some brown squirrels in a tree. Then, in both movies, some black squirrels run up the tree. Both movies end with 13 squirrels in the tree—some black, and some brown. The way to model each of these problems is to start with the scene at the end of the movie.

Step	Add To/Change Unknown	Add To/Start Unknown
1	Draw the 13 squirrels in the tree.	Draw the 13 squirrels in the tree.
2	Show the 5 squirrels that are brown.	Show the 5 squirrels that are black.
3	Count the squirrels that are black.	Count the squirrels that are brown.

So in spite of the names of these two problem types, they are both subtraction problems—to solve them, some of the squirrels have to be taken away from the set of all 13 squirrels.

Add To/Change Unknown problems begin on page 63 and are followed by **Add To/Start Unknown** problems, beginning on page 71.

Teaching Tip 2

More Algebra – The two problems about squirrels can be represented by the equations $5 + \square = 13$ and $\square + 5 = 13$, respectively. Some children may want to think about problems like these in that way. As above, thinking in this way is a good introduction to algebra, and we believe that it should be encouraged. On the other hand, as first-graders, it is more important that they see why both problems can be solved by subtraction. Subtraction works because both involve “taking away.”

Compare/Difference Unknown

There are 13 brown squirrels.

There are 5 black squirrels.

How many more brown squirrels are there than black squirrels?

Help children to model this problem by following these steps. Their drawing should show 18 squirrels.

1. Draw the 13 brown squirrels.
2. Draw the 5 black squirrels.
3. Show which of the brown squirrels match the 5 black squirrels.
4. Count the extra brown squirrels.

Compare/Difference Unknown problems begin on page 79.

Compare/Smaller Unknown

There are 13 brown squirrels.

There are 5 more brown squirrels than black squirrels.

How many black squirrels are there?

To understand that this is a subtraction problem, children need to see that some of those 13 brown squirrels correspond to the black squirrels and that the rest of those brown squirrels are “extra.” To find the number of black squirrels, they need to “take away” those 5 extras. You can help them by teaching them to model the problem with these steps.

1. Draw the 13 brown squirrels.
2. Show which of those brown squirrels are the 5 extra ones.
3. Count the remaining brown squirrels. (There are 8.)
4. Draw the 8 black squirrels.

Teaching Tip 3


Why Is This Subtraction? Steps 2 and 3 are very important. When children show which of the brown squirrels are “extra,” and when they count the remaining brown squirrels, they are helped to think about why this is a subtraction problem. Their pictures should help them to see that we are “taking away” 5 from 13.

Compare/Smaller Unknown problems begin on page 87.

Take From/Change Unknown - 1

Name _____

There are 12 chickens.
Some of them go away.
There are 8 chickens left.
How many chickens went away? 4



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This child started with a very simple representation of the 12 chickens. To find how many went away, she circled the 8 chickens that did not go away.

Name _____

There are 10 chickens.

5 of them go away.

How many chickens are there now? _____

Name _____

There are 8 boys.

5 of them are eating.

The rest are sleeping.

How many boys are sleeping? _____