UNIT 1

Introduction to Algebra Tiles

Unit Overview

Areas of squares and rectangles are key to naming and working with Algebra Tile pieces. Beginning with the small yellow/red squares, it is common to denote the length of each side as 1. Therefore, the area is 1 square unit. The value of a yellow square is 1, whereas the value of a red square is –1. The blue/red square customarily has sides of length x. Its area is x^2 square units. The value of a blue square is x^2, whereas the value of a red square is $-x^2$. The dimensions of a green/red rectangle are 1 unit by x units, and its area is x square units. Activities in the unit first involve representing integers using the small squares, and then proceed to representing and writing algebraic expressions using the Algebra Tiles. The Zero Principle is introduced and used throughout this unit and future units. Note: Work mat is on page 9.

Glossary of Terms

Integers: The set of numbers {. . . –3, –2, –1, 0, 1, 2, 3, . . .}

Opposites: Two numbers that have the same absolute value but opposite signs.

Zero Pair: Two tiles of the same size, one of each color. A zero pair sums to zero.

Variable: A symbol, usually a letter, that is used to represent one or more numbers in an algebraic expression. For example, x is a variable in the expression $4x + 3$.

Polynomial: An expression that has one or more terms of the form ax^n where a is any real number and n is a whole number.

Binomial: A polynomial that has two terms.

Trinomial: A polynomial that has three terms.

Term: A part of an expression that is separated by an addition or subtraction sign. In the expression $2x - 4$, the terms are $2x$ and –4.

Activity 1.1 Integer Pieces and the Zero Principle

Objectives: To represent positive and negative integers using Algebra Tiles. To represent the opposite of an integer using Algebra Tiles. To represent zero pairs using Algebra Tiles. To model integers using zero pairs.

Prerequisites:

A. Students need to know that the area of a square is given by $A = s^2$, where s is the length of a side.

B. Students need to be able to represent positive and negative integers using Algebra Tiles.

Getting Started:

A. Introduce the unit squares. Discuss the idea of unit length, so that the area of the square is 1 square unit. Discuss the idea of a negative integer. Note that we can use the yellow unit squares to represent positive integers and the red unit squares to represent negative integers.

B. Show students how two small squares of opposite colors (yellow and red) neutralize each other, so that the net result of such a pair is zero.

C. A positive integer can be represented by other positive integers in many ways.

For example, 6 = 3 + 3 = 5 + 1 = 4 + 2.

Similarly, positive and negative integers can be represented by other positive and negative integers in many ways. Start with a positive integer such as 2. Add zero pairs and discuss the new name for 2. Continue this process several times. Repeat the activity; however, begin with a negative integer and add zero pairs.

Closing the Activity:

A. Present verbal descriptions to students and have them model the results with Algebra Tiles.

B. Present various collections of unit squares, positive and negative, and have students use the Zero Principle to simplify the collection. Given that there are an odd number of unit squares, have students determine possible outcomes: all yellow, positive integer; all red, negative integer; more yellow than red, positive integer; more red than yellow, negative integer.

C. Have students sketch more than one model for a given positive or negative integer. Give students an illustration of several positive and negative integers using Algebra Tiles, and have them name the integer represented in the model.

Answers:

1. 6 yellow squares
2. 4 red squares
3. Answers will vary.
4. 2 yellow squares
5. 10 red squares
6. 5 red squares
7. $4 + (-4) = 0$; $7 + (-7) = 0$
8. (a) 2; $4 + (-2) = 2$; (b) -6; $(-7) + 1 = -6$; (c) 4; $6 + (-2) = 4$; (d) 3; $6 + (-3) = 3$; (e) -4; $(-6) + 2 = -4$;(f) 1; $(-3) + 4 = 1$
9. (a)–(f) Teacher check.
10. Infinite. Starting with 4 yellow tiles, as many zero pairs can be added as you wish.
11. Infinite. Starting with 3 red tiles, as many zero pairs can be added as you wish.

Activity 1.2 Naming Algebra Tile Pieces

Objective: To name the remaining Algebra Tile pieces (x, x^2).

Prerequisites: An area model is used in naming Algebra Tile pieces. Students must know that the area of a square is given by $A = s^2$, where s is the length of a side. Students must know that the area of a rectangle is given by $A = lw$, where l is the length and w is the width of the rectangle.

Getting Started: Using a document camera, show the other Algebra Tile pieces. Show that the length of a side of the blue/red square is not an integer value. We will say that the length is x. Therefore, the area is x^2 square units. The blue square has a value of x^2, whereas the red square has a value of $-x^2$. The green Algebra Tile piece has a value of x, and the red Algebra Tile piece has a value of $-x$. Using a document camera and a collection of Algebra Tiles, write an algebraic expression for the collection. Then write an algebraic expression and have students model it with the tiles.

Closing the Activity: The activity concludes with change in the length of a side of the small square. With a side of length y, the area now becomes y^2 square units. The value of a yellow square is y^2, and the value of a small red square is $-y^2$. Expressions involving two variables can be written.

Answers:

1.–4. Teacher check

5. $x^2 + 3xy + 5y^2$
6. $2x^2 + (-2xy) + y^2$
7. $-3x^2 + 4xy + (-3y^2)$
8. 4 blue a^2 pieces, 2 red ab pieces, 2 yellow b^2 pieces

Activity 1.3 Expression Grab

Objective: To practice writing and simplifying algebraic expressions, using the Zero Principle.

Prerequisites: Students must know how to apply the Zero Principle in order to simplify algebraic expressions.

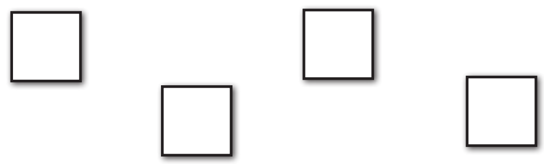
Getting Started: Using Algebra Tiles with a document camera, illustrate how to simplify an algebraic expression, using the Zero Principle where necessary.

Closing the Activity: Show illustrations of Algebra Tile pieces and have students write a simplified expression. Write algebraic expressions and have students write simplified expressions.

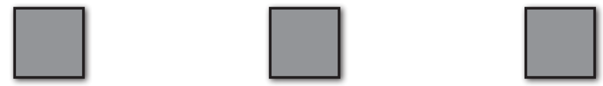
ACTIVITY 1.1 Integer Pieces and the Zero Principle

Name:

To represent integer values, use the small yellow and red square tiles.



Each yellow square represents +1.



Each red square represents –1.

Use the small square tiles to represent each number. Make a sketch of your model.

1. 6	2. –4
3. A single digit odd number	4. An even prime number
5. 10 feet below sea level	6. 5 degrees below zero

The numbers +1 and –1 *are called* ***opposites****. They sum to* 0*. That is,* 1 + (–1) = 0*. We refer to* 1 *and* –1 *as a* ***zero pair****. In an elevator, for example, if you go up one floor* (+1) *and then down one floor* (–1)*, you are back to where you started.*

When we add positive and negative integers, we often use what is called the ***Zero Principle*** *to perform the addition operation.*

The mats below show two basic examples of the *Zero Principle*. We make pairs of yellow and red tiles.

1 + (–1) = 0

2 + (–2) = 0

UNIT 2

Integer Operations

Unit Overview:

In Unit 2, Algebra Tiles are used to model integer arithmetic for addition, subtraction, and multiplication. Ideas about zero pairs from Unit 1 are incorporated in **Activities 2.1–2.10.** Subtraction of integers is related to addition of integers. Games are included that provide practice on integer addition and subtraction. Patterns or rules for each operation are highlighted. For multiplication of positive and negative integers, the generalized form $a \times b$ in terms of "a" groups of "b" squares is presented. Note: Work mat is on page 9.

Glossary of Terms:

Commutative Property of Addition: For any integers a and b, $a + b = b + a$.

Activity 2.1 Modeling Addition

Objective: To model addition of positive and negative integers using Algebra Tiles.

Prerequisites: Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: Pose basic addition problems such as 2 + 3 = ? and use Algebra Tiles to model them. Then pose addition problems involving negative integers and explain how Algebra Tiles can be used to model and solve these problems using zero pairs.

Closing the Activity: Pose addition problems different from those on the sheet. Have individual students model and solve the problems with Algebra Tiles on their mats, or by using a document camera if available.

Answers:

1. 3 + (–2) = 1
2. (–4) + (–1) = –5
3. (–5) + 4 = –1

4.–6. Teacher check

Activity 2.2 Addition of Integers

Objective: To add positive and negative integers. To use additive inverses and the additive identity to find sums of positive and negative integers.

Prerequisites: Students must be able to model addition of positive and negative integers using Algebra Tiles.

Getting Started: Review the additive identity property for addition of whole numbers. Review the idea of the opposite or additive inverse of an integer.

Closing the Activity: Review the additive identity property for addition of integers. Discuss the sign patterns involved in adding integers:

> The sum is positive if the signs of both integers are positive.
>
> The sum is negative if the signs of both integers are negative.
>
> The sum is positive if the positive integer is greater than the absolute value of the negative integer.
>
> The sum is negative if the absolute value of the negative integer is greater than the positive integer.

Present problems illustrating each case. Have students model them with Algebra Tiles and determine the sums.

Answers:

1. 0
2. 0
3. 0
4. –5
5. –3
6. 8
7. opposites
8. the number
9. 9
10. –8
11. 12
12. –10
13. 12
14. –13
15. Suggested Answer: Use the sign of the two numbers and add the absolute values.

16. 633
17. –655
18. –365
19. 3
20. 4
21. 2
22. 4
23. 3
24. 3
25. Answers will vary: When there are more yellow pieces than red pieces the answer will be positive.
26. –5
27. –8
28. –3
29. –4
30. –5
31. –3
32. Answers will vary: When there are more red pieces than yellow pieces, the answer will be negative.
33. –
34. +
35. –
36. Answers will vary: If there are more yellow pieces, the sum is positive. If there are more red pieces, the sum is negative. In each case, remove the zero pairs and find the number of pieces that remain.
37. 35
38. 133
39. –6

40.–43. Teacher check

Activity 2.3 Three Addends

Objective: To find the sum of three or more integers.

Prerequisites: Students must be able to find the sum of two integers.

Getting Started: Provide an example for addition with three positive integers, showing that we can add any two at a time to find the sum. Then, provide examples with three addends where one or more of the addends are negative integers.

Closing the Activity: When adding three or more integers, we can add two at a time in any order. The patterns or rules established in Activity 2.2 then apply.

Answers:

1. 3
2. –11
3. –1
4. 11
5. –1
6. –2
7. Answers will vary.
8. 188
9. –143

Activity 2.4 The Game of One

Objective: To provide practice on adding positive and negative integers in a game format.

Prerequisites: Students must know how to add positive and negative integers. Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: To play the Game of One each student needs 11 unit squares, the game mat, and a Game of One Record Sheet.

Closing the Activity: Have students play several games, closest to 1. Change rules so that the player with the greatest sum (or least sum) is the winner.

Activity 2.5 Modeling Subtraction

Objective: To model subtraction of two integers using Algebra Tiles.

Prerequisites: Students must understand the Zero Principle and using Algebra Tiles to form zero pairs.

Getting Started: Model basic subtraction problems with Algebra Tiles. Then model a subtraction problem where the known subtrahend is a negative integer, such as 2 – (–3).

Discuss adding zero pairs to the model.

Closing the Activity: Subtraction of two integers using Algebra Tiles employs the "take–away" language from primary grades. If it is impossible to "take–away" a positive integer or a negative integer, then zero pairs must used.

Answers:

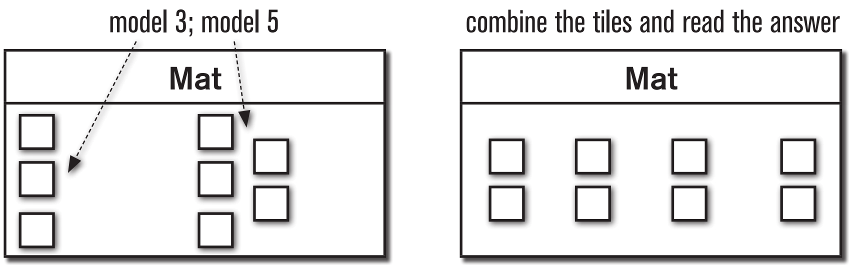
1. 6 – 3 = 3
2. –8 – (–5) = –3
3. –5 – (–4) = –1
4. 6 red squares with 4 circled to take away; –2
5. 3 red squares with 2 circled to take away; –1
6. 5 yellow squares with 3 circled to take away; 2

Modeling Addition

Name:

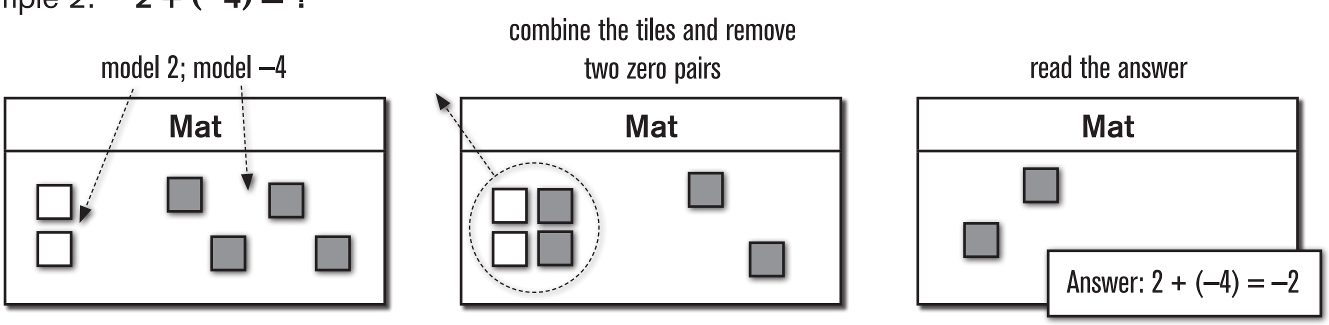
To add two integers, model each quantity on the work mat, combine the Algebra Tiles, remove any zero pairs, and read the answer.

Example 1: **3 + 5 = ?**



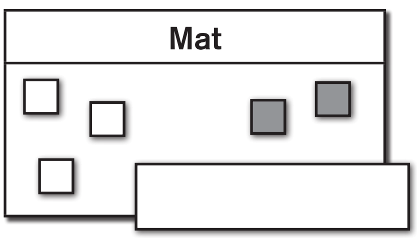
Answer: 3 + 5 = 8

Example 2: **2 + (−4) = ?**

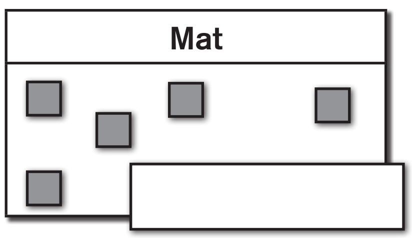


Write the addition problem modeled in each diagram. Use Algebra Tiles to determine the sum.

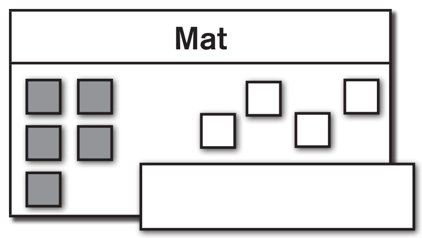
1.



2.

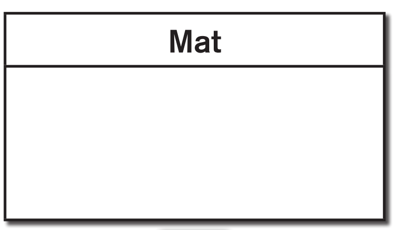


3.

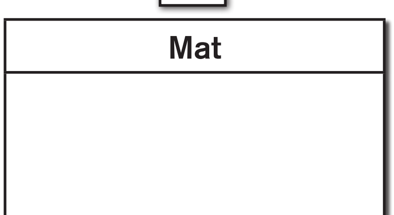


Use Algebra Tiles to determine the sum. Draw a diagram to model each addition problem. Circle any zero pairs. Use the second work mat to draw a diagram of the sum and record your answer.

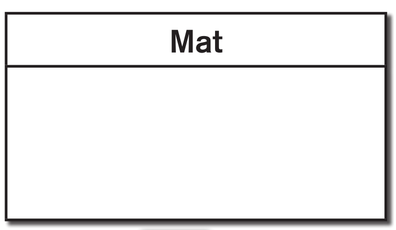
4. 4 + (−3)



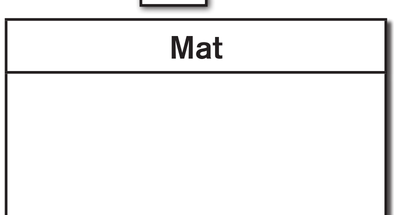
4 + (−3) = ☐



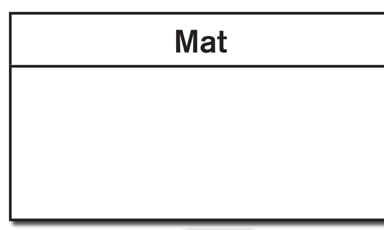
5. −6 + 3



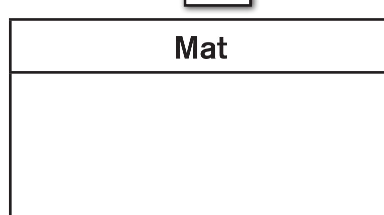
−6 + 3 = ☐



6. −3 + (−5)



−3 + (−5) = ☐



Addition of Integers

Name:

Use the Algebra Tiles and work mat to find the following sums.

1. $4 + (-4) =$ ______ 2. $-2 + 2 =$ ______ 3. $6 + (-6) =$ ______
4. $0 + (-5) =$ ______ 5. $-3 + 0 =$ ______ 6. $8 + 0 =$ ______

Concept Summary

7. If the sum of two integers is zero, then the two numbers must be ______________________.
8. The sum of zero and any number is ______________________.

Use the Algebra Tiles and work mat to find the following sums.

9. $6 + 3 =$ ______ 10. $-5 + (-3) =$ ______ 11. $7 + 5 =$ ______
12. $-6 + (-4) =$ ______ 13. $8 + 4 =$ ______ 14. $-4 + (-9) =$ ______

Concept Summary

15. To add two integers with the same sign, ______________________

______________________.

Use the rule you have written to find the following sums.

16. $415 + 218 =$ ______ 17. $-395 + (-260) =$ ______ 18. $-247 + (-118) =$ ______

Now consider the sum of two integers with different signs. Use the Algebra Tiles and work mat to find the following sums.

19. $5 + (-2) =$ ______ 20. $-5 + 9 =$ ______ 21. $6 + (-4) =$ ______
22. $-6 + 10 =$ ______ 23. $-2 + 5 =$ ______ 24. $8 + (-5) =$ ______
25. Describe any patterns that you observed in answering Questions 19 to 24. ______________________

____

____

Try a few more addition problems involving the sum of two integers with different signs. Use the Algebra Tiles and work mat to find the following sums.

26. $4 + (-9) =$ ______ 27. $-11 + 3 =$ ______ 28. $5 + (-8) =$ ______
29. $-10 + 6 =$ ______ 30. $-9 + 4 =$ ______ 31. $7 + (-10) =$ ______

Linear Expressions

Unit Overview:

Activities in the first two units have focused on introducing Algebra Tile pieces and modeling integer operations with the tiles. Unit 3 is an extension of Unit 2; it provides activities that involve algebraic expressions–those that use the Algebra Tiles x-pieces.

Algebra vocabulary/terminology is highlighted in **Activity 3.1.** Students continue work with terminology in **Activity 3.2**, first spinning two monomials, then writing a symbolic and written expression for that result. Making matches between models and algebraic expressions is the focus of bingo and concentration games in **Activity 3.3.** Additionally, students find the additive inverses of algebraic expressions. To evaluate a linear expression using Algebra Tiles, the expression is modeled and then each x–tile is replaced with its value.

Activity 3.4 leads students through cases in which x is positive and ones in which x is negative. Closest to Zero in **Activity 3.5** is a game for students to practice evaluating linear expressions. Students reach into a bag and grab Algebra Tiles, and then evaluate the corresponding expression with a given x-value. The distributive property developed in elementary and middle schools is extended to algebraic expressions in **Activity 3.6.** The focus in **Activity 3.7** is on adding like or similar terms, using the Zero Principle to simplify sums.

Activity 3.8 involves using Binomial Expression Cards, where students draw cards from a deck, model the linear expressions with Algebra Tiles, and simplify to find the sum. Finally, in **Activity 3.9,** students use Algebra Tiles to subtract linear expressions. The lesson contains two methods. In the first, students use the "take–away" model from elementary school, and in the second, they use the idea presented in Unit 2 that a subtraction problem can be rewritten as an addition problem. Note: Work mat is on page 9.

Glossary of Terms:

Distributive Property: For all integers a, b, and c, $a(b + c) = ab + ac$.

Like or Similar Terms: Terms with identical variable parts raised to the same power

Activity 3.1 Translating Algebraic Expressions

Objective: To translate from a word description of an algebraic expression to a model using Algebra Tiles.

Prerequisites: Students must know the names of the Algebra Tiles pieces. Students must be able to write an algebraic expression.

Getting Started: Provide an algebraic expression, such as $3x - 2$, for students to discuss. Write various word descriptions for the expression:
Three times a number x decreased by 2
A multiple of 3 minus 2
The difference between three times a number and 2
Three times a number less 2

Closing the Activity: Write an algebraic expression on the board or use a document camera. First, have a student model the expression with Algebra Tiles. Have students provide verbal descriptions of the expression. Second, write a verbal description on the board or use a document camera, then have students at their seats model the expression with tiles and write the corresponding algebraic expression.

Answers:

1. $2x - 1$
2. $-5 + 4x$
3. $2x + 3$
4. $-x + 4$
5. $-3x + 5$
6. $3x - 1$

Activity 3.2 Spin an Expression

Objective: To practice writing a word description for an algebraic expression expressed in symbols.

Prerequisites: Students must be able to model addition of positive and negative integers using Algebra Tiles.

Getting Started: Provide each student with a copy of the Algebra Tiles Spinner Sheet, a spinner, and Activity 3.2. Discuss the directions with students and provide an example such as $2x - 3$. Have students express word descriptions for the expression.

Closing the Activity: Review the additive identity property for addition of integers. Discuss the sign patterns involved in adding integers:

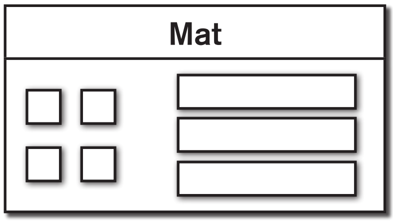
- sum is positive if the signs of both integers are positive.
- The sum is negative if the signs of both integers are negative.

Translating Algebraic Expressions

Name:

Below are two different word descriptions of algebraic expressions and a representation of each using Algebra Tiles.

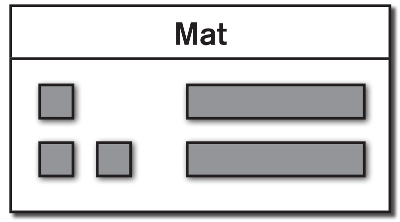
Three times a number increased by 4



What is an algebraic expression for this model?

____

Two times the opposite of a number decreased by 3



What is an algebraic expression for this model?

____

Below are several word descriptions of algebraic expressions. On your work mat, use Algebra Tiles to model each expression. Sketch your model in the corresponding mat. Then write an algebraic expression for your model.

1. Two times a number x decreased by 1

   Mat

2. Opposite of 5, increased by four times a number x

   Mat

3. Three added to the double of a number x

   Mat

4. The opposite of x increased by 4

   Mat

5. Five added to the opposite of three times a number x

   Mat

6. Triple of a number x decreased by 1

   Mat

7. Add an x-tile to each side of the equation. Add 3 to each side of the equation. Simplify, and find the answer.
8. $x = -1$
9. Add two x-tiles to each side of the equation. Remove zero pairs and $3 = 5 + 2x$. Add -5 to each side of the equation. Remove zero pairs and $-2 = 2x$. Separate each side of the equation into two equal groups and $-1 = x$.
10. $x = -2$
11. $x = 4$
12. $x = 1$

Activity 4.3 Solving Linear Equations with Variables on Both Sides

Objective: To solve linear equations of the form: $ax + b = cx + d$.

Prerequisites: Students must know how to solve linear equations of the form $ax + b = c$, where a is positive or negative.

Getting Started: Illustrate an equation with variables on both sides of the equal sign. Discuss the idea of getting all variables on one side and all constants on the other side of the equal sign.

Closing the Activity: Provide another linear equation with variables on both sides of the equal sign. Have students explain how to solve the equation. Provide a linear equation where the solution is not an integer.

Answers:

1. $x = -3$
2. $x = 2$
3. $x = 4$
4. $x = 1$
5. $x = 5$
6. $x = 2$
7. $x = 3$
8. $x = 1$
9. $x = -4$
10. Add 3 to each side of the equation. Remove zero pairs and $4x = 3x + 2$. Subtract $3x$ from each side of the equation. Solve and $x = 2$.
11. Add 3 x-tiles to each side of the equation and get $4x - 4 = 0$. Add 4 to each side of the equation. Remove zero pairs and $4x = 4$. Separate each side of the equations into 4 equal groups and $x = 1$.

Activity 4.4 Solving Linear Inequalities

Objective: To solve linear inequalities of the form $ax + b < c$ or $ax + b > c$.

Prerequisites: Students must know how to solve linear equations of the form $ax + b = c$. Students must know how to find integer solutions to inequalities of the form $x < b$ or $x > b$.

Getting Started: Solve inequalities of the form $x < b$ or $x > b$. Point out that solutions to linear inequalities often differ from those to linear equations. Sometimes there will be no integer value that satisfies the inequality; other inequalities provide one or more solutions. Note that the procedures for solving linear inequalities with Algebra Tiles are the same as solving linear equations. The difference comes in finding the integer values in the set being considered that satisfy the inequality.

Closing the Activity: In using Algebra Tiles to solve linear inequalities, point out that only integer solutions are considered. When different replacement sets are considered, solutions might be other types of numbers, such as rational numbers or real numbers.

Use as an example the last inequality on the activity page, $x < 4$. Note that numbers like 1/2, 3, 5/7 and others also satisfy the inequality.

Answers:

1. {2, 3, 4}
2. {0, 1}
3. None
4. $x < 5$; {0, 1, 2, 3, 4}
5. $x < 2$; {0, 1}
6. $x < 1$; {0}
7. $x < 1$; {0}
8. $x < -1$; {}
9. $x < 2$; {0, 1}
10. $x > 6$; {7, 8}
11. $x > -1$; {0, 1, 2, 3, 4, 5, 6, 7, 8}
12. $x < 1$; {0}
13. Add 6 to each side of the inequality. Remove zero pairs and $4x < 8$. Separate each side of the inequality into 4 equal groups and $x < 2$. From the set of numbers provided, the solution set is {0, 1}.

Activity 4.5 Solving Linear Inequalities (Coefficient of *x*-term is negative)

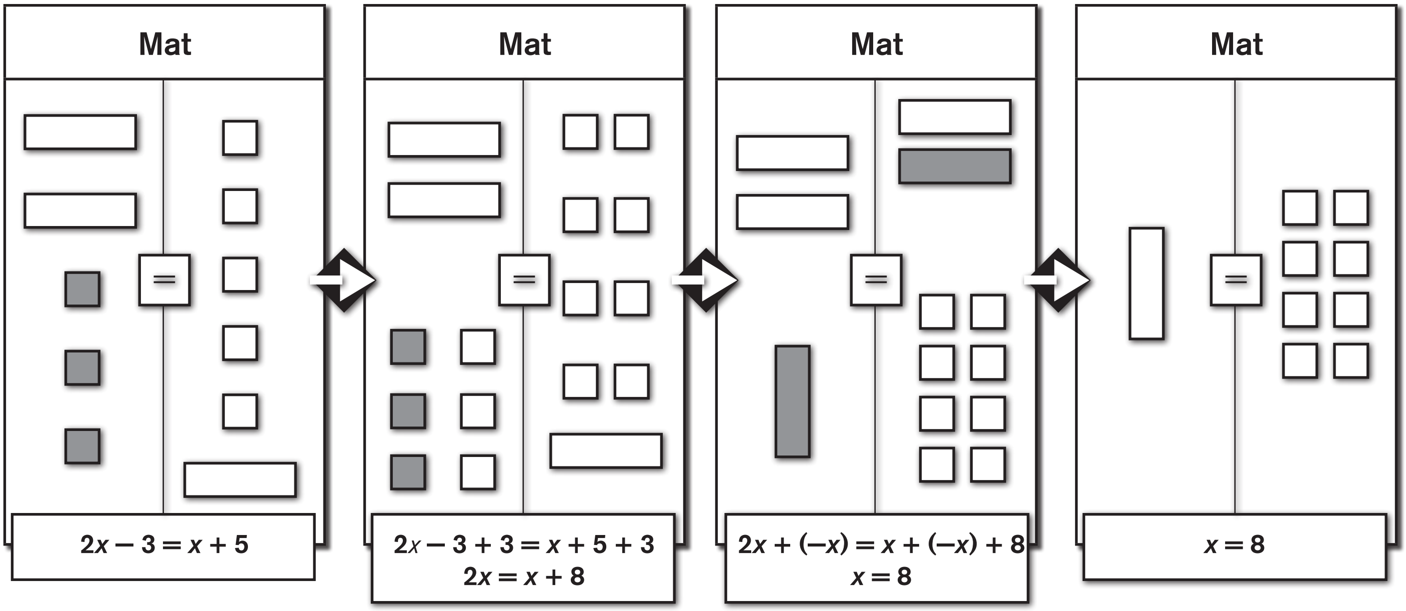
Objective: To solve linear inequalities of the form $ax + b < c$ or $ax + b > c$, where a is negative.

Prerequisites: Students must know how to solve linear inequalities of the form $ax + b < c$ or $ax + b > c$, where a is positive.

ACTIVITY 4.3 **Solving Linear Equations – Variables on Both Sides**

Name:

Often linear equations have variables on both sides of the equal signs. For example, $2x - 3 = x + 5$. *Again, we can use Algebra Tiles and apply the Zero Principle to solve this equation.*



Use Algebra Tiles and a work mat to solve the following linear equations.

1. $2x - 1 = 3x + 2$ __________ 2. $-x + 4 = -2x + 6$ __________ 3. $3x + 3 = 4x - 1$ __________

4. $-2x + 3 = -x + 2$ __________ 5. $-3x + 3 = -2x - 2$ __________ 6. $4x - 3 = 3x - 1$ __________

7. $x + 4 = 3x - 2$ __________ 8. $x - 4 = -3x$ __________ 9. $2x - 3 = 4x + 5$ __________

10. Explain the steps you used to find the value of x in Problem 6. ____________________

Problems 1 through 6 are all similar; however, problems 7, 8, and 9 have a slight variation.

11. Explain the steps you used to find the value of x in Problem 8. ____________________