SECTION 1

Integers, Variables, and Expressions

Teaching Notes 1.1: Using the Order of Operations

The order of operations is a set of rules for simplifying expressions that have two or more operations. A common mistake students make is to perform all operations in order from left to right, regardless of the proper order.

1. Present this problem to your students: $10 - 3 \times 2 + 2 \div 2$. Ask your students to solve. Some will apply the correct order of operations and find that the answer is 5, which is correct. Others will solve the problem in order from left to right and arrive at the answer of 8. Explain that this is the reason we use the order of operations. It provides rules to follow for solving problems.
2. Explain that to simplify an expression, the order of operations must be followed. State the following rules for the order of operations:
   - Perform all multiplication and division in order from left to right.
   - Perform all addition and subtraction in order from left to right.

   Emphasize that multiplication and division must be done first, no matter where these symbols appear in the expression.
3. Provide some examples, such as those below. Ask your students what steps they would follow to simplify the expressions. Then ask them to simplify each example.

$3 + 2 \times 5 \div 5$	Steps: $\times, \div, +$	Answer is 5.
$15 \div 5 \div 3 \times 2$	Steps: $\div, \div, \times$	Answer is 2.
$12 - 2 \div 2 \times 8$	Steps: $\div, \times, -$	Answer is 4.

4. Review the information and examples on the worksheet with your students.

EXTRA HELP:

Be sure you have performed all of the operations in their proper order.

ANSWER KEY:

(1) 5 **(2)** 21 **(3)** 4 **(4)** 18 **(5)** 24 **(6)** 188 **(7)** 1 **(8)** 39 **(9)** 8 **(10)** 48 **(11)** 10 **(12)** 16

(Challenge) The subtraction symbol should be inserted in the blank.

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WORKSHEET 1.1: USING THE ORDER OF OPERATIONS

Mathematicians have agreed to simplify expressions that have no exponents or grouping symbols according to the following rules:

1. Multiply and divide in order from left to right.
2. Start at the left again and add and subtract from left to right.

EXAMPLES

$50 - 3 \times 2 \times 5 =$	$10 + 2 - 3 \times 4 =$	$32 \div 2 - 2 \times 3 =$
$50 - 6 \times 5 =$	$10 + 2 - 12 =$	$16 - 2 \times 3 =$
$50 - 30 =$	$12 - 12 =$	$16 - 6 =$
20	0	10

DIRECTIONS: Simplify each expression.

1. $12 - 2 \times 4 + 1$
2. $12 \times 4 \div 2 - 3$
3. $10 \times 2 - 2 \times 8$
4. $10 \times 2 - 6 \div 3$
5. $8 + 1 + 6 \times 5 \div 2$
6. $48 \div 2 \times 8 - 4$
7. $15 - 2 - 2 \times 6$
8. $35 + 8 - 12 \div 3$
9. $3 \times 7 - 8 - 5$
10. $20 \times 2 + 10 - 4 \div 2$
11. $8 - 4 + 2 \times 3$
12. $40 \div 8 - 5 + 3 \times 2 + 10$

CHALLENGE: Place the correct operation symbol in the blank so that 12 ___ $3 \times 2 + 2 = 8$.

Teaching Notes 1.2: Simplifying Expressions That Have Grouping Symbols

If an expression contains grouping symbols, the order of operations requires that whatever part of the expression is contained in the grouping symbols be simplified first. A common mistake of students is to ignore the grouping symbols when simplifying.

1. Explain that grouping symbols are sometimes used to enclose an expression. There are several types of grouping symbols, including parentheses, brackets, and the fraction bar. Parentheses are the most common.
2. Explain the meaning of grouping symbols. For example, $3 \times (4 + 2)$ means 3 groups of 6 which equals 18. Emphasize that this is quite different from $3 \times 4 + 2$, which means 3 groups of 4 plus 2 more and is equal to 14. Have your students solve each problem. Discuss why each provides a different answer.
3. Explain that all operations within parentheses should be done first, following the order of operations.
4. Review the steps for the order of operations and the examples on the worksheet with your students.

EXTRA HELP:

The multiplication sign is often omitted before a grouping symbol. Example: $3(5 + 4)$ is the same as $3 \times (5 + 4)$.

ANSWER KEY:

(1) 36 **(2)** 2 **(3)** 52 **(4)** 3 **(5)** 104 **(6)** 4 **(7)** 0 **(8)** 22

(Challenge) Yes. The parentheses are not necessary. Each expression equals 3.

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WORKSHEET 1.2: SIMPLIFYING EXPRESSIONS THAT HAVE GROUPING SYMBOLS

Common grouping symbols include parentheses ( ), brackets [ ], and the fraction bar—. Follow the steps below to simplify expressions with grouping symbols:

1. Simplify expressions within grouping symbols first by following the order of operations. Multiply and divide in order from left to right. Then add and subtract in order from left to right.
2. After you have simplified all expressions within grouping symbols, multiply and divide in order from left to right.
3. Add and subtract in order from left to right.

EXAMPLES

$14 - (8 - 6) \times 5 =$	$2[3 + 5 \times 4 - 1] =$	$1 + \frac{3 + 12}{3} =$
$14 - 2 \times 5 =$	$2[3 + 20 - 1] =$	$1 + \frac{15}{3} =$
$14 - 10 =$	$2[23 - 1] =$	$1 + 5 =$
4	$2[22] = 44$	6

DIRECTIONS: Simplify the following expressions.

1. $(8 + 4) \times 3$
2. $28 \div (7 \times 2)$
3. $80 - 2[6 + 4 \times 2]$
4. $(3 + 6) - (2 + 4)$
5. $12 + 4(8 + 3 \times 5)$
6. $\frac{15 - 1}{7} \times 2$
7. $3(4 + 6) - 5(10 - 4)$
8. $15 + \frac{2 + 6 \times 2}{2}$

CHALLENGE: Is $3 \times 2 - 12 \div 4$ the same as $(3 \times 2) - (12 \div 4)$? Explain your reasoning.

Teaching Notes 1.3: Simplifying Expressions with Nested Grouping Symbols

If an expression has nested grouping symbols—one or more sets of grouping symbols inside another—some students ignore the innermost symbols. They then go on to simplify the expression incorrectly.

1. Explain that grouping symbols help to indicate what operations to do first when solving expressions.
2. Explain that when a grouping symbol is set within another, the expression within the innermost grouping symbol must be simplified first. Provide the following example: $12 - (5 + (3 \times 2))$. Explain that there are two sets of parentheses in this problem. Operations in the inner set of parentheses must be completed first and then work is completed outward. Demonstrate this by first solving 3×2 and replacing the answer, 6, in the new problem: $12 - (5 + 6)$. The correct answer is 1.
3. Emphasize that students should always work outward from the nested grouping symbol, following the order of operations. Depending on your students, you may want to review the order of operations:
   - Multiply and divide from left to right.
   - Add and subtract from left to right.
4. Review the steps for simplifying and the examples on the worksheet with your students. Note the use of grouping symbols and particularly the innermost grouping symbols.

EXTRA HELP:

Parentheses, brackets, and fraction bars are examples of grouping symbols.

ANSWER KEY:

(1) 24 **(2)** 22 **(3)** 120 **(4)** 80 **(5)** 91 **(6)** 220 **(7)** 4 **(8)** 3

(Challenge) 4

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WORKSHEET 1.3: SIMPLIFYING EXPRESSIONS WITH NESTED GROUPING SYMBOLS

Sometimes an expression has one or more grouping symbols inside another. These are often called "nested" grouping symbols. Follow the steps below when simplifying expressions with nested grouping symbols:

1. Simplify the expressions within the nested grouping symbols first.
2. After simplifying the innermost expression, work outward.
3. Simplify the expression according to the order of operations. Multiply and divide from left to right. Then add and subtract from left to right.

EXAMPLES

$20 - [3 \times (14 - 12)] =$	$4[(6 + 3) \times 10] =$	$\frac{3 + 24}{12 - (10 - 7)} =$
$20 - [3 \times 2] =$	$4(9 \times 10) =$	$\frac{3 + 24}{12 - 3} =$
$20 - 6 =$	$4(90) =$	$\frac{27}{9} =$
14	360	3

DIRECTIONS: Simplify.

1. $4[9 - (5 - 2)]$
2. $2[4 + 7(4 - 3)]$
3. $2[3(12 - 7) \times 4]$
4. $4[8(6 - 3) - 4]$
5. $3 + 2[4(3 + 8)]$
6. $(3 + 2)[4(3 + 8)]$
7. $\frac{2(3 + 7)}{3 + 2}$
8. $\frac{6 \times 8}{2(3 + 5)}$

CHALLENGE: What is the missing number? $6 + [\_\_\_(3 + 3(8 + 1))] = 126$

Teaching Notes 1.4: Using Positive Exponents and Bases Correctly

Many students make mistakes when working with positive exponents and bases. One of the most common errors is equating x^n with $x \times n$.

SPECIAL MATERIALS

Graph paper

1. Explain that an exponent represents the number of times a base is used as a factor. For example, $5^2 = 5 \times 5$ and $5^3 = 5 \times 5 \times 5$. Emphasize that 5^2 does not equal 5×2 or 2×5 and 5^3 does not equal 5×3 or 3×5.
2. Ask your students to draw a square, five units per side, on graph paper.
3. Instruct them to count the number of small squares inside the large square.
4. Explain that they should count twenty-five small squares. These squares represent 5×5 or 5^2. Emphasize that 5 is a factor two times, which is the meaning of 5^2, pronounced "five squared." It is termed "squared" because when modeled geometrically 5^2 forms a square. This may help your students remember that it is 5 times 5, not 5 times 2. Likewise, 5 to the third power is often called "five cubed." When modeled geometrically, 5^3 forms a cube with five units on each edge.
5. Next ask your students to draw a rectangle, five units long and two units wide, on graph paper. They should count ten small squares inside the rectangle. These squares represent 5×2, which is quite different from 5^2.
6. Review the examples on the worksheet with your students. Emphasize that in the first example 4 is a factor 3 times. In the second example 3 is a factor 5 times.

EXTRA HELP:

x^n means x is a factor n times.

ANSWER KEY:

(1) 16, 8 **(2)** 18, 81 **(3)** 81, 12 **(4)** 12, 64 **(5)** 21, 343 **(6)** 100, 20 **(7)** 10, 32 **(8)** 1, 3 **(9)** 25, 10 **(10)** 18, 216

(Challenge) 2, because $2^2 = 4$ and $2 \times 2 = 4$.

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WORKSHEET 1.4: USING POSITIVE EXPONENTS AND BASES CORRECTLY

An exponent indicates the number of times its base is used as a factor. In 3^2, 3 is the base and 2 is the exponent.

EXAMPLES

$4^3 = 4 \times 4 \times 4 = 64$ $3^5 = 3 \times 3 \times 3 \times 3 \times 3 = 243$

DIRECTIONS: Find the value of each expression.

1. 2^4 4×2
2. 2×9 9^2
3. 3^4 4×3
4. 2×6 2^6
5. 3×7 7^3
6. 10^2 10×2
7. 5×2 2^5
8. 1^3 3×1
9. 5^2 5×2
10. 6×3 6^3

CHALLENGE: $x^2 = 2x$ is true for only one positive integer. What number makes this equation true? Explain how you know your answer is correct.

Teaching Notes 1.5: Simplifying Expressions with Grouping Symbols and Exponents

Expressions that involve exponents, parentheses, or several operations are often confusing to students. To ensure that your students become proficient in simplifying such expressions, reinforcement of the order of operations is essential.

1. Explain that some expressions contain exponents. Depending on the abilities of your students, you might find it helpful to review 1.4: "Using Positive Exponents and Bases Correctly."
2. Explain to your students that the order of operations may become confusing when they must compute using multiple operations. Suggest that students use the acronym "Please excuse my dear Aunt Sally" to help them remember the order of operations for expressions with grouping symbols and exponents:
   - P stands for parentheses (or grouping symbols).
   - E stands for exponents.
   - M stands for multiplication.
   - D stands for division.
   - A stands for addition.
   - S stands for subtraction.

   Note that although multiplication precedes division in the acronym, these operations must be completed in order from left to right. Therefore, there will be times students will divide before multiplying. Similarly, addition precedes subtraction in the acronym, and these operations must also be completed in order from left to right. There will be times students will subtract before adding.
3. Review the steps for using the order of operations and the examples on the worksheet with your students.

EXTRA HELP:

Suggest that students rewrite each problem after they have completed an operation. This will help them organize their work and avoid mistakes.

ANSWER KEY:

(1) 13 **(2)** 112 **(3)** 43 **(4)** 19 **(5)** 46 **(6)** 26 **(7)** 3 **(8)** 3

(Challenge) 7

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WORKSHEET 1.5: SIMPLIFYING EXPRESSIONS WITH GROUPING SYMBOLS AND EXPONENTS

To simplify expressions with grouping symbols, exponents, and other operations, follow the steps below:

1. Simplify expressions within grouping symbols first. Simplify the innermost expressions first and continue working outward to the outermost expressions. As you do, be sure to follow steps 2, 3, and 4.
2. Simplify powers.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

EXAMPLES

$3 + 4 \times 2 + 5^2 =$	$4^2 + (3 + 5) \times 2 =$	$3^2[(3 + 4) \times 2] =$
$3 + 4 \times 2 + 25 =$	$4^2 + 8 \times 2 =$	$3^2[(7 \times 2)] =$
$3 + 8 + 25 =$	$16 + 8 \times 2 =$	$3^2 \times 14 =$
$11 + 25 =$	$16 + 16 =$	$9 \times 14 =$
36	32	126

DIRECTIONS: Simplify.

1. $2 \times 3^2 - 5$

2. $37 + 5^2 \times 3$

3. $2^3 + 7(8 - 3)$

4. $(2 + 1)^3 - 2^3$

5. $(13 - 2^3) \times 2 + 6^2$

6. $2^2 \times 8 - 5 - 1$

7. $\frac{48}{2^4}$

8. $\frac{(5 - 2)^3}{2 \times 7 - 5}$

CHALLENGE: What is the missing number? $\frac{4(\_\_\_\_ + 3)^2}{5(14 - 3^2)} = 16$

Teaching Notes 1.6: Evaluating Expressions

Evaluating an expression requires students to replace each variable in an expression with a given value and simplify the result. Common errors occur when students either substitute an incorrect value for the variable or follow the order of operations incorrectly.

1. Review variables by explaining that a variable represents an unknown quantity. It is usually expressed as a letter.
2. Explain that sometimes students are required to find a variable's value. At other times the value of a variable is provided. When the value of a variable is provided, students must replace the variable in the expression with that value.
3. Stress to your students the importance of substituting values for variables correctly.
4. Encourage them to rewrite the problem after they have substituted the correct values.
5. Review the order of operations and examples on the worksheet with your students. Caution them to pay close attention to nested grouping symbols. Depending on their abilities, you may find it helpful to review 1.3: "Simplifying Expressions with Nested Grouping Symbols."

EXTRA HELP:

A number directly before a variable denotes multiplication. For example, $3a$ means 3 times a.

A number or variable above or below a fraction bar denotes division. For example, $\frac{a}{4}$ means a number divided by 4.

ANSWER KEY:

(1) 7 **(2)** 58 **(3)** 56 **(4)** 64 **(5)** 2 **(6)** 29 **(7)** 14 **(8)** 26

(Challenge) Answers may vary. One acceptable response is $c(d - a) - b$.

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WORKSHEET 1.6: EVALUATING EXPRESSIONS

To evaluate an expression means to replace a variable or variables with a given number or numbers and then simplify the expression. Follow the steps below:

1. Rewrite the expression by replacing all the variables with the given values. Be sure you have substituted correctly.
2. Follow the order of operations for simplifying:
   - Simplify expressions within grouping symbols first. If there are nested grouping symbols, simplify the innermost first, then work outward.
   - Simplify powers.
   - Multiply and divide in order from left to right.
   - Add and subtract in order from left to right.

EXAMPLES

$a = 3, b = 4, c = 5$, and $d = 6$.

$2a + 10c =$	$2a^2 + cd =$	$2(b + c^2) =$
$2 \times 3 + 10 \times 5 =$	$2 \times 3^2 + 5 \times 6 =$	$2(4 + 5^2) =$
$6 + 50 =$	$2 \times 9 + 5 \times 6 =$	$2(4 + 25) =$
56	$18 + 30 =$	$2(29) =$
	48	58

DIRECTIONS: Evaluate each expression if $a = 3, b = 4, c = 5$, and $d = 6$.

1. $ab - c$

2. $8d + 2c$

3. $8(a + b)$

4. $(a + c)^2$

5. $\frac{ab}{d} =$

6. $c(a + b) - d$

7. $d + 2[a + (c - b)]$

8. $\frac{cd}{a} + b^2$

CHALLENGE: Use the values and variables above to create an expression that equals 11.

Teaching Notes 1.7: Writing Expressions

Writing expressions is a prerequisite skill to writing equations. Most of the errors students make in writing expressions arise from misinterpreting words and phrases, particularly those having to do with subtraction and division.

1. Explain that key words often signal addition, subtraction, multiplication, and division. Following are some examples:
   - *Addition:* add, total, in all, combine, sum, increased by
   - *Subtraction:* less than, more than, subtract, difference, decreased by
   - *Multiplication:* product, multiply, of, twice, double, triple
   - *Division:* divide, quotient, split, groups of, quarter
2. Direct your students to focus their attention on subtraction and division. Point out that unlike addition and multiplication, subtraction and division are not commutative; the proper order of the terms cannot be switched.
3. Provide the following example: 4 less than a number n. Ask your students to write an expression for this phrase, then discuss the answer. Explain that although 4 comes first in the phrase, it must be placed after the n in the expression. The correct expression for the phrase 4 less than n is $n - 4$. It cannot be $4 - n$. Offer this illustration: $6 - 4 \neq 4 - 6$.
4. Provide this example: A number n divided by 2. Ask your students to write an expression for this phrase. It is $n \div 2$. Note that it cannot be $2 \div n$. Offer this illustration: $4 \div 2 \neq 2 \div 4$.
5. Review the chart on the worksheet with your students. You might ask your students to generate more examples.

EXTRA HELP:

To check your work when writing expressions, pick a number, substitute it for the variable, and see if the result is reasonable.

ANSWER KEY:

(1) $n + 6$ or $6 + n$ **(2)** $n - 1$ **(3)** $3n$ **(4)** $n - 8$ **(5)** $n \div 10$ **(6)** $n - 9$ **(7)** $n - 6$ **(8)** $3 + n$ or $n + 3$

(Challenge) Answers may vary. An acceptable response is 3 less than twice a number.

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WORKSHEET 1.7: WRITING EXPRESSIONS

The chart below illustrates some expressions and examples of their meanings. n represents a number.

ADDITION: $n + 3$	SUBTRACTION: $n - 3$
a number increased by 3	a number decreased by 3
3 more than a number	3 less than a number
a number plus 3	a number minus 3
the sum of a number and 3	a number reduced by 3
MULTIPLICATION: $2n$	DIVISION: $n \div 2$
the product of 2 and a number	a number divided by 2
2 times a number	half of a number
2 multiplied by a number	the quotient when a number is divided by 2

DIRECTIONS: Write an expression for each phrase. Use n to represent the number.

1. 6 greater than a number
2. A number minus 1
3. 3 times a number
4. 8 less than a number
5. A number divided by 10
6. A number decreased by 9
7. A number minus 6
8. The sum of a number and 3

CHALLENGE: Write a phrase for this expression: $2n - 3$.

Teaching Notes 1.8: Writing Expressions Involving Grouping Symbols

Some expressions describe operations as a sum, difference, product, or quotient. To write expressions like these, students may have to include grouping symbols. Ignoring necessary grouping symbols is a common error.

1. Discuss basic examples of expressions:
   - 3 times a number plus 2 can be written as $3n + 2$.
   - 3 times the sum of a number and 2 can be written as $3(n + 2)$.
2. Explain that in the first example the number is multiplied by 3, then 2 is added. In the second example, the sum of the number and 2 is multiplied by 3. $n + 2$ must be written in parentheses.
3. Emphasize that these two expressions have different values. For example, if $n = 4$, $3n + 2 = 14$ and $3(n + 2) = 18$.
4. Encourage your students to consider whether an expression refers to a quantity or only one number. Remind them that the words "sum," "difference," "product," and "quotient" often signify that grouping symbols are needed.
5. Review the examples on the worksheet with your students.

EXTRA HELP:

Quantities must be written in grouping symbols.

ANSWER KEY:

(1) $\frac{n+12}{5}$ **(2)** $n + \frac{12}{5}$ **(3)** $(4n)^3$ **(4)** $4n^3$ **(5)** $2(n - 10)$ **(6)** $2n - 10$ **(7)** $(4 + n)^2$ **(8)** $4 + n^2$

(Challenge) $\frac{x+y}{2}$

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WORKSHEET 1.8: WRITING EXPRESSIONS INVOLVING GROUPING SYMBOLS

An expression for a quantity that is added, subtracted, multiplied, or divided must be written within a grouping symbol. Key words such as ''sum,'' ''difference,'' ''product,'' and ''quotient'' often indicate two or more numbers that are usually written within grouping symbols.

EXAMPLES

3 times a number squared: $3n^2$. Only the number, n, is squared.
The product of 3 times a number, squared: $(3n)^2$. The quantity, $3n$, is squared.

DIRECTIONS: Write an expression for each phrase. Use n to represent a number.

1. The sum of a number and 12, divided by 5
2. A number plus 12 divided by 5
3. The product of 4 times a number, cubed
4. 4 times a number cubed
5. Twice the difference when 10 is subtracted from a number
6. Two times a number minus 10
7. The sum of 4 and a number, squared
8. 4 plus a number squared

CHALLENGE: Write an expression to show the average of x and y.

Teaching Notes 1.9: Identifying Patterns by Considering All of the Numbers

Identifying patterns and recognizing the relationship between numbers is an important skill. Students frequently make a quick decision based on examining only a few numbers in a pattern instead of all of them, which, of course, leads to mistakes.

1. Explain that the numbers in any pattern are related in some way.
2. Emphasize that students must consider all, not merely some, of the numbers of a pattern before they can identify the relationship between the numbers in the pattern.
3. Provide your students with the following example: 2, 4, 8, . . .
4. Ask them to consider only the first two numbers. Explain that two relationships for the numbers are possible:
   - Each number is two more than the previous one.
   - Each number is two times the previous one.
5. Direct your students to now consider the third number. Explain that the third number proves that the only correct relationship is that each number is two times the number before it.
6. Review the information and examples on the worksheet with your students.

EXTRA HELP:

All numbers in a pattern must be related.

ANSWER KEY:

(1) 3 is subtracted from the previous number.
(2) 8 is added to the previous number.
(3) The second number is found by adding 10, the third is found by adding 1, and this pattern continues.
(4) Numbers are found by adding 3, then adding 4, adding 5, and so on.
(5) Each number is the square of the previous number.
(6) Each number is one-third of the previous number.
(7) Each number is 3 more than the previous number.
(8) Each number is found by subtracting 1, then adding 1, and so on.
(9) Each number is a multiple of 4.
(10) Each number is found by dividing the previous number by 5.

(Challenge) The missing number is 11. Each number is the sum of the two preceding numbers.

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WORKSHEET 1.9: IDENTIFYING PATTERNS BY CONSIDERING ALL OF THE NUMBERS

A pattern is a group of numbers that are related in some way. Following are some hints for recognizing patterns:

- If the numbers are increasing, consider the operations of addition and multiplication.
- If the numbers are decreasing, consider the operations of subtraction and division.
- Some number patterns involve a combination of operations.

EXAMPLES

4, 6, 8, 10, . . . Each number is 2 more than the preceding number.
100, 50, 25, 12.5, . . . Each number is half of the preceding number.
10, 19, 37, 73, . . . Each number is found by multiplying the previous number by 2 and subtracting 1.

DIRECTIONS: Identify the pattern in each sequence below.

1. 64, 61, 58, 55, . . .

2. 2, 10, 18, 26, . . .

3. 2, 12, 13, 23, 24, . . .

4. 3, 6, 10, 15, 21, . . .

5. 5, 25, 625, . . .

6. 729, 243, 81, 27, . . .

7. 0, 3, 6, 9, . . .

8. 19, 18, 19, 18, . . .

9. 4, 8, 12, 16, . . .

10. 1,000, 200, 40, . . .

CHALLENGE: What is the missing number in this pattern?
3, 4, 7, ___, 18, 29, 47, . . .

Teaching Notes 1.10: Writing Prime Factorization

Expressing a composite number as a product of its prime factors is called writing the prime factorization. A common error students make is failing to complete the prime factorization of a number by mistaking a composite number for a prime.

1. Make sure your students understand the definition of a prime number. A prime number is a whole number greater than 1 that has only two factors: 1 and itself. Compare this definition with that of a composite number: a whole number greater than 1 that has more than two factors. Note that 1 is neither prime nor composite.
2. Point out the first ten prime numbers on the worksheet. Ask your students to volunteer examples of composite as well as other prime numbers.
3. Explain that every composite number can be uniquely expressed as the product of prime factors. The order of the prime factors does not matter. $15 = 3 \times 5$ or 5×3.
4. Review the information and example on the worksheet with your students. Note that all of the factors in prime factorization are prime numbers and that the factors are written in ascending order. Also note how the prime factors can be written with exponents. For example, the prime factorization of 20 can be written as $2 \times 2 \times 5$ or $2^2 \times 5$.

EXTRA HELP:

2 is a prime factor of every even number.

ANSWER KEY:

(1) $2 \times 5 \times 7$ **(2)** $2^2 \times 7$ **(3)** $2^2 \times 3 \times 5$ **(4)** $2^2 \times 5^2$ **(5)** 3^4 **(6)** $3^2 \times 5$
(7) 3×5^2 **(8)** $2^3 \times 3$ **(9)** $2^2 \times 3 \times 5^2$ **(10)** $2 \times 3^2 \times 7$

(Challenge) $6 = 2 \times 3$. 2 and 3 are the two smallest prime numbers.

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WORKSHEET 1.10: WRITING PRIME FACTORIZATION

Every composite number can be written as a product of prime numbers. This is called ''prime factorization.'' To write the prime factorization of a composite number, follow these steps:

1. Find two factors of the composite number.
2. Factor each of the factors (if possible).
3. Continue this process until all of the factors are prime numbers.

Here are the first ten prime numbers: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29.

EXAMPLE

Find the prime factorization of 200. (*Note:* In the example, the prime numbers that are used to write the prime factorization are underlined.)

- Two factors of 200 are 10 and 20. $10 \times 20 = 200$.
- The prime factors of 10 are 2 and 5. $\underline{2} \times \underline{5} = 10$.
- Two factors of 20 are 4 and 5. $4 \times \underline{5} = 20$. 5 is a prime number but 4 is a composite number and must be factored.
- The two prime factors of 4 are 2 and 2. $\underline{2} \times \underline{2} = 4$.
- Write the prime factorization by writing all of the prime factors. Therefore, the prime factorization of $200 = 2 \times 2 \times 2 \times 5 \times 5$ or $2^3 \times 5^2$.

DIRECTIONS: Find the prime factorization of each number.

1. 70 **2.** 28 **3.** 60 **4.** 100 **5.** 81

6. 45 **7.** 75 **8.** 24 **9.** 300 **10.** 126

CHALLENGE: What is the smallest number that can be expressed as the product of two different prime numbers?

Teaching Notes 1.11: Finding the Greatest Common Factor

When students are asked to find the greatest common factor (GCF) of two numbers via the method of prime factorization, confusion may arise over the use of the smallest exponent of a common base or bases. Because they are finding the "greatest" common factor, some students mistakenly believe they must use the greatest exponent of a common base or bases.

1. Explain that the greatest common factor is the largest common factor of two or more numbers.
2. Instruct your students to find the greatest common factor of 315 and 135. Work through the process together.
3. Ask for volunteers to provide the prime factorization of 315 and 135:
   - $315 = 3 \times 3 \times 5 \times 7$
   - $135 = 3 \times 3 \times 3 \times 5$
4. Ask your students to list the factors that are common to 315 and 135, and then find the product. $3 \times 3 \times 5 = 45$. The GCF of 315 and 135 is 45.
5. Explain that prime factorization with exponents can be used to find the GCF. Note that 315 and 135 have a common factor of 3^2. They also have a common factor of 5. Therefore, $3^2 \times 5 = 45$, which is the GCF.
6. Review the information and example on the worksheet with your students and emphasize why $2^2 \times 3$ is the GCF.

EXTRA HELP:

If there are no common factors listed in the prime factorization of two numbers, the GCF is 1. Although 1 is not a prime number, it is a factor of every number.

ANSWER KEY:

(1) 3 **(2)** 5 **(3)** 6 **(4)** 12 **(5)** 6 **(6)** 10 **(7)** 5 **(8)** 20 **(9)** 5 **(10)** 1 **(11)** 2 **(12)** 9

(Challenge) Disagree. Explanations may vary. One explanation is that 1 is the GCF of any two prime numbers. But 1 may also be the GCF of two numbers that are composite. For example, 1 is the GCF of 4 and 15. Also, 1 may be the GCF of a prime and composite number. For example, 1 is the GCF of 3 and 10.

Name ______________________________ Date ______________

WORKSHEET 1.11: FINDING THE GREATEST COMMON FACTOR

The greatest common factor (GCF) is the greatest factor that two (or more) numbers have in common. To find the GCF of two numbers, follow the steps below:

1. Use exponents to write the prime factorization of each number.
2. Find the prime numbers that are factors of both numbers.
3. Choose the smaller exponent of each common prime factor. The base with the smaller exponent is always a factor of the same base with a larger exponent.
4. The product of the common factors is the GCF.

EXAMPLE

Find the GCF of 72 and 60.

$72 = 2^3 \times 3^2$ $\quad\quad 60 = 2^2 \times 3 \times 5$

Common factors are 2 and 3.
2 is the smaller exponent of 2. (2^2 is a factor of 2^3.)
1 is the smaller exponent of 3. (3^1 is a factor of 3^2.)
The GCF of 72 and 60 $= 2^2 \times 3 = 12$.

DIRECTIONS: Find the GCF of each pair of numbers.

1. 9 and 12
2. 25 and 35
3. 6 and 60
4. 36 and 192
5. 36 and 30
6. 90 and 400
7. 135 and 50
8. 100 and 140
9. 45 and 200
10. 67 and 9
11. 48 and 50
12. 9 and 72

CHALLENGE: Do you agree with the following statement? If the GCF of two numbers is 1, then both numbers must be prime. Explain your answer.

Teaching Notes 1.12: Finding the Least Common Multiple

Students often confuse the least common multiple (LCM) with the greatest common factor (GCF). Reinforcing the meaning of factors, multiples, and common multiples can reduce confusion.

1. Discuss the meaning of the greatest common factor (GCF) of two numbers: the largest factor that two numbers have in common. For example, the GCF of 8 and 12 is 4.
2. Illustrate the concepts of multiples and the least common multiple by showing multiples of 30 and 12:
   - Ask your students to list multiples of 30: 30, 60, 90, 120, 150, 180, 210, . . .
   - Ask them to list multiples of 12: 12, 24, 36, 48, 60, 72, 84, 96, . . .
3. Ask your students to identify the first number that is common to both lists, which is 60. 60 is the least common multiple of 30 and 12. (You might mention that if the lists were extended more common multiples would be found.) Note that this is a somewhat tedious method, especially for large numbers.
4. Explain to your students that rather than list multiples to find the LCM of two numbers, they can use prime factorization. Instruct them to write the prime factorization of 30 and 12. $30 = 2 \times 3 \times 5$ and $12 = 2^2 \times 3$. They can find the LCM by finding the product of each base raised to the highest power of each prime factor. The LCM of 30 and 12 is $2^2 \times 3 \times 5 = 60$.
5. Review the information and example on the worksheet with your students.

EXTRA HELP:

The LCM of two numbers will always be greater than or equal to the larger number.

ANSWER KEY:

(1) 60 **(2)** 120 **(3)** 80 **(4)** 720 **(5)** 108 **(6)** 90 **(7)** 75 **(8)** 448 **(9)** 345 **(10)** 240 **(11)** 24 **(12)** 1,430

(Challenge) The statement is true. Explanations may vary. An acceptable response is if 1 is the only common factor the LCM is found by multiplying all of the numbers in the prime factorization. *Example:* 15 and 14 have no common factors other than 1. $15 = 3 \times 5$ and $14 = 2 \times 7$. Therefore the LCM of 15 and 14 is $2 \times 3 \times 5 \times 7 = 210$.

Name ______________________________ Date ______________

WORKSHEET 1.12: FINDING THE LEAST COMMON MULTIPLE

The least common multiple (LCM) is the smallest number that is a multiple of two or more numbers. To find the LCM of two numbers, do the following:

1. Write the prime factorization of each number.
2. Find the product of each prime number raised to the largest power.

EXAMPLE

Find the LCM of 24 and 180.

$24 = 2^3 \times 3$ $\quad\quad$ $180 = 2^2 \times 3^2 \times 5$

In the prime factorization of 24 and 180, the largest power of 2 is 2^3 and the largest power of 3 is 3^2. The only power of 5 is 5^1, which is 5. The LCM of 24 and 180 is $2^3 \times 3^2 \times 5 = 360$.

DIRECTIONS: Find the LCM of each pair of numbers.

1. 15, 12 **2.** 20, 24 **3.** 16, 10 **4.** 45, 48

5. 27, 36 **6.** 18, 90 **7.** 25, 75 **8.** 64, 28

9. 115, 15 **10.** 6, 80 **11.** 4, 24 **12.** 11, 130

CHALLENGE: Do you agree with the following statement? If two numbers have no common factor other than 1, then their product is the LCM. Explain your answer.

Teaching Notes 1.13: Classifying Counting Numbers, Whole Numbers, and Integers

Although there are several types of numbers, students often have trouble classifying three of the most common: counting numbers, whole numbers, and integers. Many students simply lump these three types of numbers together.

1. Ask for a volunteer to give an example of the counting numbers. Most likely the student will say 1, 2, 3, 4, 5, . . . Write the list on the board and explain that the ellipsis means that the counting numbers continue in this manner, adding 1 to the previous number. They go on infinitely. Offer this suggestion to your students to help them remember that the counting numbers begin with 1: When you start counting something, you start with 1.
2. Explain that zero is not a part of the counting numbers. Zero and the counting numbers make up the whole numbers. 0, 1, 2, 3, 4, 5, . . . Mention to your students that "whole" has an "0," which can be thought to represent "zero."
3. Explain that zero is necessary to some applications of math, as are negative numbers. For example, on the Celsius scale, a subfreezing temperature on a cold winter day might be -5° or five degrees less than zero. The counting numbers, zero, and the opposites of the counting numbers are integers. . . . −3, −2, −1, 0, 1, 2, 3, . . .
4. Review the examples of counting numbers, whole numbers, and integers on the worksheet with your students. Note that the counting numbers are a subset of the whole numbers and the whole numbers are a subset of the integers.

EXTRA HELP:

Think of whole numbers as the counting numbers and zero.

ANSWER KEY:

(Counting numbers) 15, 1, 4, 12, 3, 23, 154, 752, 19 (Whole numbers) 15, 1, 0, 4, 12, 3, 23, 154, 752, 19
(Integers) 15, −8, 1, 0, −10, −7, 4, 12, 3, −1, 23, 154, −30, 752, 19

(Challenge) Answers may vary. Possible answers include below zero temperatures, a loss of yardage in a football game, a decrease in business activity, or the deduction of points on a test.

Name ______________________________ Date ______________

WORKSHEET 1.13: CLASSIFYING COUNTING NUMBERS, WHOLE NUMBERS, AND INTEGERS

Numbers can be classified into various groups. Three of the most important groups are shown below:

- *Counting numbers:* These are the numbers we count with. Starting with 1, they continue forever by adding 1 to the previous number: 1, 2, 3, 4, 5, . . .
- *Whole numbers:* Zero and the counting numbers make up the whole numbers: 0, 1, 2, 3, 4, 5, . . .
- *Integers:* Negative and positive whole numbers and zero make up the integers: . . . −3, −2, −1, 0, 1, 2, 3, . . .

Most numbers belong to more than one of the groups above. For example, 3 is a counting number, a whole number, and an integer. But −3 is only an integer.

DIRECTIONS: Classify the numbers below as counting numbers, whole numbers, and integers. Some numbers may be members of more than one group.

15	−8	1	0	−10
−7	4	12	3	−1
23	154	−30	752	19

CHALLENGE: Give at least three examples of how negative numbers may be used.

Teaching Notes 1.14: Finding Absolute Values and Opposites

Two terms associated with integers are "absolute value" and "opposite." These terms are easily confused. Sometimes the opposite of an integer and its absolute value are the same; sometimes they are different.

1. Explain that the absolute value of a number is the number of units the number is from 0 on the number line. Offer this example: the absolute value of both −2 and 2 is 2, because both numbers are 2 units from 0.
2. Model absolute value on the number line. Show that −2 is two spaces from 0. Therefore, its absolute value is 2. Show that 3 is three spaces from 0. Therefore, its absolute value is 3. Remind students that the absolute value of a number is always positive. This is because it represents the distance from 0. Distances cannot be negative.
3. Explain that the opposite of a number always has the opposite sign, except for 0, which is neither positive nor negative. The opposite of −8 is 8 and opposite of 3 is −3. Note that the sum of opposite integers is always 0.
4. Review the number line and information on the worksheet with your students. Point out the following:
    - Every integer is the same distance from the number to its right or left.
    - Positive numbers are to the right of zero.
    - Negative numbers are to the left of zero.

EXTRA HELP:

Negative numbers may be written with a lowered minus sign, −7, or a raised minus sign, $^{-}7$.

ANSWER KEY:

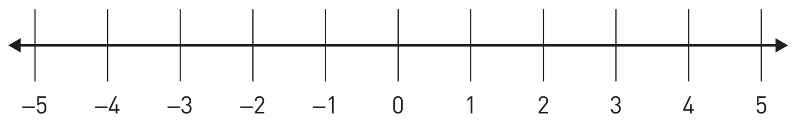
(1) −6 **(2)** 4 **(3)** 1 **(4)** 5 **(5)** 10 **(6)** 12 **(7)** 20 **(8)** 8 and −8

(Challenge) Explanations may vary. Absolute value is the number of units a number is from zero on the number line. Opposite numbers are the same distance from zero, one to the right, the other to the left.

Name ______________________________ Date ______________

WORKSHEET 1.14: FINDING ABSOLUTE VALUES AND OPPOSITES

A number line is a way to picture numbers and points on a graph.



Each pair of numbers that are the same distance from 0 are opposites. For example, 1 is the opposite of −1 and −3 is the opposite of 3. The absolute value of a number is the number of units the number is from 0 on the number line. The direction does not matter.

EXAMPLES

$|2|$ (read "the absolute value of 2") = 2 because 2 is 2 units from 0.

$|-2| = 2$ because −2 is 2 units from 0.

$|0| = 0$ because 0 is 0 units from 0.

The opposite of −7 is 7; the opposite of 5 is −5.

DIRECTIONS: Find the integer or integers described below.

1. The opposite of 6
2. The absolute value of −4
3. The absolute value of 1
4. The opposite of −5
5. The opposite of −10
6. The absolute value of 12
7. The opposite of −20
8. Two numbers whose absolute value is 8

CHALLENGE: Explain how opposites and absolute values both relate to distance.

Teaching Notes 1.15: Adding Integers with Different Signs

Adding two integers with different signs may seem like subtraction of integers to some students. They do not realize that when adding a positive integer and a negative integer, the negative integer "negates" the value of the positive integer.

1. Ask your students to imagine that a student gained 50 points while playing a video game and then lost 10 points. Explain that the loss of 10 points negates 10 of the 50-point gain. The student now has only 40 points. The 10-point loss can be represented by -10.
2. Explain that this situation can be modeled by adding integers. $50 + (-10) = 40$. The process is addition because the student gained 50 *and* lost 10. The word "and" indicates addition.
3. Now ask your students to suppose that another student lost 40 points and gained 30 in a video game. The 40-point loss has gained 30 more points, resulting in only a 10-point loss. This can also be modeled by adding integers. $-40 + 30 = -10$
4. Review the rules and examples for adding integers on the worksheet with your students.

EXTRA HELP:

Positive symbols for numbers can be omitted as a number without a sign is assumed to be positive. Negative signs must always be included.

ANSWER KEY:

(1) 17 **(2)** 9 **(3)** 0 **(4)** -8 **(5)** -7 **(6)** -24 **(7)** 62 **(8)** 49 **(9)** 5 **(10)** -25 **(11)** 14 **(12)** -12

(Challenge) Answers may vary. One acceptable response is $-10 + 3 = -7$ and $5 + (-4) = 1$.

Name ______________________________ Date ______________

WORKSHEET 1.15: ADDING INTEGERS WITH DIFFERENT SIGNS

Use the following rules to add a positive and negative integer:

1. Find the absolute value of each number.
2. Subtract the smaller absolute value from the larger.
3. Keep the sign of the integer with the greater absolute value.

EXAMPLES

$-3 + 15$	$18 + (-25)$
$\lvert -3 \rvert = 3 \quad \lvert 15 \rvert = 15$	$\lvert 18 \rvert = 18 \quad \lvert -25 \rvert = 25$
$15 - 3 = 12$	$25 - 18 = 7$
Because $\lvert 15 \rvert > \lvert -3 \rvert$, the sign is positive.	Because $\lvert -25 \rvert > \lvert 18 \rvert$, the sign is negative.
$-3 + 15 = 12$	$18 + (-25) = -7$

DIRECTIONS: Find each sum.

1. $-12 + 29$
2. $-1 + 10$
3. $8 + (-8)$
4. $-15 + 7$
5. $5 + (-12)$
6. $-38 + 14$
7. $98 + (-36)$
8. $-26 + 75$
9. $20 + (-15)$
10. $-36 + 11$
11. $22 + (-8)$
12. $-44 + 32$

CHALLENGE: Create two problems that involve adding a positive and a negative integer so that one sum is positive and the other is negative. Include the answers to your problems.

Teaching Notes 1.16: Subtracting Integers

Subtraction of integers can be easy for students to master if they remember to rewrite the subtraction problem as an addition problem and then add the integers. Unfortunately, forgetting to rewrite the problem is a very common mistake.

1. Explain to your students that subtracting a number is defined by adding its opposite. $a - b = a + (-b)$, if a and b are integers.
2. Explain that any subtraction problem can be rewritten as an addition problem. It is often easier to add integers than subtract them.
3. Review the information and examples on the worksheet with your students. Encourage them to rewrite subtraction problems involving integers by following the steps provided. If necessary, review the steps for adding integers provided with 1.15: "Adding Integers with Different Signs."

EXTRA HELP:

The opposite of a negative number is a positive number and the opposite of a positive number is a negative number.

ANSWER KEY:

(1) −7 **(2)** −9 **(3)** 15 **(4)** 28 **(5)** 16 **(6)** −5 **(7)** −15 **(8)** 0 **(9)** −11 **(10)** 35 **(11)** −14 **(12)** −2

(Challenge) Rewrite the problem as $-3 - [-10 + (-6)]$. Find the sum of −10 and −6, which is −16. Rewrite the subtraction problem as addition: $-3 - (-16) = -3 + 16$. The sum is 13.

Name ______________________________ Date ______________

WORKSHEET 1.16: SUBTRACTING INTEGERS

You can subtract integers by rewriting the subtraction problem as addition. Follow the steps below:

1. Change the subtraction sign to addition.
2. Change the sign of the second number to its opposite.
3. Follow the rules for adding integers.

EXAMPLES

−3 − 15 =	6 − 10 =	−5 − (−20) =	8 − (−2) =
−3 + (−15) =	6 + (−10) =	−5 + 20 =	8 + 2 =
−18	−4	15	10

DIRECTIONS: Find each difference.

1. 8 − 15
2. 3 − 12
3. 11 − (−4)
4. 12 − (−16)
5. 8 − (−8)
6. 0 − 5
7. −6 − 9
8. 8 − 8
9. −7 − 4
10. 20 − (−15)
11. −18 − (−4)
12. 9 − 11

CHALLENGE: Write the steps for solving the following subtraction problem. Then find the difference. −3 − (−10 − 6)

Teaching Notes 1.17: Multiplying Two Integers

A common problem students have when multiplying integers is not recognizing that the product of two negative numbers is a positive number. Students may then incorrectly write a negative sign instead of the positive sign in the product.

1. Explain that the product of a positive integer and a negative integer is a negative integer. To illustrate, offer the following pattern:
   - $-1 \times 3 = -3$
   - $-1 \times 2 = -2$
   - $-1 \times 1 = -1$
   - $-1 \times 0 = 0$
   - $-1 \times (-1) = 1$
   - $-1 \times (-2) = 2$
   - $-1 \times (-3) = 3$

   Explain that as the multiplier is decreasing by 1, the product is increasing by 1. The pattern continues and each answer is one more than the one before it. Emphasize that a negative integer multiplied by a negative integer results in a positive product.
2. Review the rules and examples for multiplying integers on the worksheet with your students. If necessary, review absolute values with your students in 1.14: "Finding Absolute Values and Opposites."

EXTRA HELP:

Multiplication is commutative; numbers can be multiplied in any order.

ANSWER KEY:

(1) 32 **(2)** −20 **(3)** 6 **(4)** −18 **(5)** 63 **(6)** −84 **(7)** 80 **(8)** −56 **(9)** −27 **(10)** 60 **(11)** 4 **(12)** −48

(Challenge) Disagree. Explanations may vary. The statement is true sometimes but not always. For example, the statement is true if two integers with the same sign are multiplied or if three positive integers are multiplied. But when three or any odd number of negative integers are multiplied it is not true.

Name ______________________ Date ______________

WORKSHEET 1.17: MULTIPLYING TWO INTEGERS

Follow the steps below to multiply two integers:

1. Multiply the absolute values of the numbers.
2. Determine the sign of the product.
   - If both integers are positive or both are negative, the product is positive.
   - If one integer is positive and the other is negative, the product is negative.
   - If one integer is zero, the product is neither positive nor negative. It is zero.

EXAMPLES

$3 \times (-5) = -15$ $-3 \times (-6) = 18$ $-4 \times 5 = -20$ $5 \times 9 = 45$

DIRECTIONS: Find each product.

1. 8×4
2. $5 \times (-4)$
3. $-3 \times (-2)$
4. -6×3
5. $-7 \times (-9)$
6. -12×7
7. $-16 \times (-5)$
8. -14×4
9. -3×9
10. $-10 \times (-6)$
11. $-2 \times (-2)$
12. $4 \times (-12)$

CHALLENGE: Do you agree with this statement? The product of two or more integers with the same sign is always positive. Explain your answer.

Teaching Notes 1.18: Multiplying More Than Two Integers

Students who possess a weak understanding of the rules for multiplying two integers will almost certainly have trouble multiplying more than two integers. The following steps can help reinforce the process.

1. Review the basic rules for multiplying two integers with your students:
   - If both integers are positive or both are negative, the product is positive.
   - If one integer is positive and the other is negative, the product is negative.
   - If one integer is zero, the product is zero. (Zero is neither positive nor negative.)
2. Explain that only two integers can be multiplied at one time. For simplicity, if three or more integers are to be multiplied, the absolute values of the first two should be multiplied and the sign should be determined. Then the absolute value of this product should be multiplied by the absolute value of the next integer and the sign should be determined. The process continues in this manner for additional integers. You may wish to note that because multiplication is commutative, integers may be multiplied in any order.
3. To reinforce how students may determine the correct sign when multiplying more than two integers, emphasize the following:
   - If all of the integers are positive, the product is positive.
   - If there is an odd number of negative integers in the problem, the product is negative.
   - If there is an even number of negative integers in the problem, the product is positive.
   - If zero is a factor, the product is zero no matter how many other factors are multiplied.
4. Review the steps for multiplying and the examples on the worksheet with your students.

EXTRA HELP:

Double-check computation and signs to correct any careless mistakes.

ANSWER KEY:

(1) −144 **(2)** 0 **(3)** 28 **(4)** 36 **(5)** −280 **(6)** −216 **(7)** 180 **(8)** −120 **(9)** −24 **(10)** −27 **(11)** 0 **(12)** 40

(Challenge) It is correct. Explanations may vary. One correct response is that the order of multiplying the factors does not affect the product. There are no mistakes in the answer.

Name ______________________________ Date ________________

WORKSHEET 1.18: MULTIPLYING MORE THAN TWO INTEGERS

To multiply more than two integers, follow the steps below:

1. Start with the first two integers and multiply their absolute values.
2. Determine the sign.
   - If the integers are both positive or both negative, the product is positive.
   - If one integer is positive and the other is negative, the product is negative.
3. Multiply the absolute value of the product of the first two integers by the absolute value of the third. Follow the same rules for finding the correct sign. Continue with this process if there are more integers to multiply.
4. If zero is a factor in the problem, the product is zero.

EXAMPLES

$-3 \times (-4) \times (-5) =$	$3 \times (-8) \times (-2) =$	$-4 \times 0 \times 5 =$
$12 \times (-5) =$	$-24 \times (-2) =$	0
-60	48	

DIRECTIONS: Find the products.

1. $6 \times (-8) \times 3$

2. $-15 \times 0 \times 9$

3. $-7 \times (-1) \times 4$

4. $-1 \times (-3) \times 12$

5. $14 \times 5 \times (-4)$

6. $-3 \times (-9) \times (-8)$

7. $15 \times 4 \times 3$

8. $-4 \times (-5) \times 6 \times (-1)$

9. $4 \times (-3) \times 2$

10. $-3 \times (-3) \times (-3)$

11. $-15 \times (-7) \times 0$

12. $5 \times (-4) \times (-2)$

CHALLENGE: Is the procedure used to multiply the following integers correct? Explain your answer. $-3 \times 4 \times (-6) \times 2 = -12 \times (-12) = 144$

Teaching Notes 1.19: Using Integers as Bases

Raising an integer to a power poses a problem for some students. These students sometimes identify the base of an expression incorrectly and then go on to simplify the expression incorrectly.

1. Explain that $(-3)^2$ is not the same as -3^2.
2. Discuss the meaning of $(-3)^2$. Note that the base, -3, is in parentheses. Explain that $(-3)^2 = -3 \times (-3) = 9$. Because -3 is a factor two times, the product is positive.
3. Discuss the meaning of -3^2. Note that in this case, 3 is the base, not -3. Explain that $-3^2 = -(3 \times 3) = -9$. Because both factors are positive and the negative sign refers to the quantity, the product is -9.
4. Explain that two different expressions may have the same value, for example:
   - $(-3)^3 = -3 \times (-3) \times (-3) = -27$ Because -3 is a factor three times, the product is negative.
   - $-3^3 = -(3 \times 3 \times 3) = -27$ Because the factors are positive and the negative sign refers to the quantity, the product is negative.
5. Review the examples on the worksheet with your students. Encourage your students to pay close attention to the base and write the problems correctly.

EXTRA HELP:

Any number, except 0, raised to the zero power is equal to 1.

ANSWER KEY:

(1) −36 **(2)** 36 **(3)** 125 **(4)** −125 **(5)** −125 **(6)** −1 **(7)** 1 **(8)** −1 **(9)** −8 **(10)** −81 **(11)** −16 **(12)** 64

(Challenge) Yes. Explanations may vary. One response is that an exponent shows the number of times the base is used as a factor. If two negative numbers are multiplied, the product is positive; if three negative numbers are multiplied, the product is negative; and so on.

Name ____ Date ________________

WORKSHEET 1.19: USING INTEGERS AS BASES

To simplify an integer raised to a power, you must first identify the base. This is particularly important if the base is a negative number.

EXAMPLES

$(-4)^2$	-4 is the base. 2 is the exponent. $(-4)^2 = -4 \times (-4) = 16$.
-4^2	4 is the base. 2 is the exponent. $-4^2 = -(4 \times 4) = -16$.
$(-4)^3$	-4 is the base. 3 is the exponent. $(-4)^3 = -4 \times (-4) \times (-4) = -64$.
-4^3	4 is the base. 3 is the exponent. $-4^3 = -(4 \times 4 \times 4) = -64$.

DIRECTIONS: Simplify each expression.

1. -6^2

2. $(-6)^2$

3. 5^3

4. -5^3

5. $(-5)^3$

6. -1^2

7. $(-1)^2$

8. $-(-1)^2$

9. -2^3

10. $-(3)^4$

11. $-(-4)^2$

12. $(-8)^2$

CHALLENGE: Is a negative number raised to an odd power always negative? Explain your answer.

Teaching Notes 1.20: Dividing Integers

Most students have little trouble dividing integers. A trouble spot for some students, however, is not understanding that dividing by zero is not possible.

1. Explain to your students that all division problems can be written in two ways. For example, $-12 \div 3$ is the same as $\frac{-12}{3}$. 3 is the divisor or denominator and -12 is the dividend or numerator. In each case, the answer, of course, is -4.
2. Explain that division can be checked by multiplication. To check that the answer to a division problem is correct, students should multiply the quotient by the divisor. The product should be the same as the dividend. $-12 \div 3 = -4$, therefore $-4 \times 3 = -12$.
3. Offer this example: $0 \div -5 = 0$. This is correct because $0 \times (-5) = 0$.
4. Now offer this example: $-5 \div 0$. Some students may say the answer is 0. Emphasize that 0 is not correct because $0 \times 0 \neq -5$.
5. Explain that you cannot divide by zero. In division, a quantity is divided into groups. But a quantity cannot be divided into zero groups. It is impossible. This is why division by zero is undefined.
6. Review the rules for determining the sign of the quotient and the examples on the worksheet with your students.

EXTRA HELP:

Use multiplication to double-check your work when you divide two integers.

ANSWER KEY:

(1) 10 **(2)** 5 **(3)** 0 **(4)** Undefined **(5)** -11 **(6)** Undefined **(7)** -24 **(8)** 0 **(9)** -9 **(10)** 21 **(11)** 4 **(12)** Undefined

(Challenge) Answers may vary. Following is a possible answer. $0 \div 3 = 0$ but $3 \div 0$ is undefined and has no solution.

Name ______________________________ Date ______________

WORKSHEET 1.20: DIVIDING INTEGERS

When dividing integers, determine the sign of the quotient by following the rules below:

1. If both the divisor and the dividend are positive or both are negative, the quotient is positive.
2. If either the divisor or the dividend is positive and the other is negative, the quotient is negative.
3. If 0 is divided by any number, the quotient is 0.
4. If any number is divided by 0, there is no solution because division by zero is undefined.

EXAMPLES

$-72 \div (-8) = 9$ $\quad \frac{-63}{9} = -7$ $\quad 40 \div 5 = 8$ $\quad 0 \div (-12) = 0$ $\quad \frac{-10}{0} =$ undefined

DIRECTIONS: Find the quotients, if possible.

1. $\frac{-100}{-10}$
2. $125 \div 25$
3. $0 \div (-18)$
4. $\frac{-20}{0}$
5. $\frac{-121}{11}$
6. $13 \div 0$
7. $144 \div (-6)$
8. $\frac{0}{29}$
9. $\frac{-108}{12}$
10. $\frac{63}{3}$
11. $\frac{-24}{-6}$
12. $\frac{-15}{0}$

CHALLENGE: Provide an example that shows that order matters when you divide two integers.

Teaching Notes 1.21: Finding Absolute Values of Expressions

To simplify an expression written within the absolute value symbol, students must think of the absolute value symbol as a grouping symbol. A common error is finding the absolute value of each number in the absolute value symbol and then finding the sum or difference.

1. Discuss the meaning of absolute value with your students. Point out that $|x|$ is the distance on the number line x is from zero. Depending on the abilities of your students, you may find it helpful to review 1.14: "Finding Absolute Values and Opposites."
2. Explain to your students that they should view absolute value as they would a grouping symbol, simplifying within it first before finding the absolute value of the number or numbers within it.
3. Offer the following examples:
   - $|3 - 12| = |3 + (-12)| = |-9| = 9$
   - $|3| - |12| = 3 - 12 = 3 + (-12) = -9$

   Emphasize that although the expressions look much alike, they are in fact quite different. In the first example, 3 and −12 are grouped within the absolute value symbol. In the second example, 3 and −12 are not grouped together but rather the difference of their absolute values must be found.
4. Review the examples on the worksheet with your students.

EXTRA HELP:

A negative symbol means the opposite. For example, $-|3|$ means the opposite of the absolute value of 3 which is −3.

ANSWER KEY:

(1) 11 **(2)** −18 **(3)** 28 **(4)** 41 **(5)** 84 **(6)** −31 **(7)** 48 **(8)** −42 **(9)** −5 **(10)** 23 **(11)** 10 **(12)** −10

(Challenge) 20 and 40.

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WORKSHEET 1.21: FINDING ABSOLUTE VALUES OF EXPRESSIONS

You can think of the absolute value symbol as a special type of grouping symbol. When working with expressions inside the absolute value symbol, you must find the absolute value of the simplified expression.

EXAMPLES

$\|3 - 10\| =$	$\|3(-6)\| =$	$\|-14 + 20\| =$	$3\|-15 + 10\| =$
$\|3 + (-10)\| =$	$\|-18\| =$	$\|6\| =$	$3\|-5\| =$
$\|-7\| =$	18	6	$3(5) =$
7			15

DIRECTIONS: Find each absolute value.

1. $|4 - 15|$

2. $-3|6 - 12|$

3. $|(-7)(-4)|$

4. $|15 + 26|$

5. $7|3(-4)|$

6. $-|15 - 46|$

7. $|3(-4)| + |2(-18)|$

8. $-3|26 - 12|$

9. $|5 - 4| - | -6|$

10. $|3| + 2| -10|$

11. $| -2 + (-4 \times 2)|$

12. $-2|1 + 4|$

CHALLENGE: Fill in the blank with a single number so that the following statement is true. There are two possible answers. Try to find both of them.
$|30 - \_\_\_\_| = 10$

Teaching Notes 1.22: Finding Square Roots of Square Numbers

When asked to find the square root of a number, some students make the mistake of dividing the number by 2. To remediate this problem, follow the procedure below.

SPECIAL MATERIALS:

Two sheets of graph paper for each student, rulers

1. Instruct your students to draw five squares: a 1×1 square, a 2×2 square, and so on up to a 5×5 square. Ask them to label two adjacent sides of each square and also write the area inside of each square. Ask them to write a number sentence to express the area of each.
2. Explain that the area of each square is equal to the length of a side squared. Thus, the area of a 3×3 square $= 9$ because $3^2 = 9$.
3. Suggest that students think of finding the square root of a number as finding the length of a side of a square. For example, $\sqrt{16} = 4$. 4 is the length of a side of a square that has an area of 16 units. Ask your students to find the square roots for each area they found (in step 1). Identifying these square roots will help them to memorize the process for finding square roots.
4. Review the list of square numbers on the worksheet with your students. Note that each of these numbers has a square root that is a positive integer. Also review the examples.

EXTRA HELP:

Square numbers are always positive.

ANSWER KEY:

(1) 3 **(2)** 6 **(3)** 10 **(4)** 9 **(5)** 1 **(6)** 2 **(7)** -5 **(8)** -1 **(9)** 11 **(10)** 10 **(11)** -3 **(12)** 2

(Challenge) $\sqrt{121} = 11$; $\sqrt{144} = 12$; $\sqrt{169} = 13$

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WORKSHEET 1.22: FINDING SQUARE ROOTS OF SQUARE NUMBERS

Think of a square number as the area of a square. The length of a side is the square root. Following is a list of the first ten square numbers:

1 4 9 16 25 36 49 64 81 100

EXAMPLE

$\sqrt{49}$ is read "the square root of 49."

$\sqrt{49} = 7$ because $7^2 = 49$.

DIRECTIONS: Find each square root.

1. $\sqrt{9}$

2. $\sqrt{36}$

3. $\sqrt{100}$

4. $\sqrt{81}$

5. $\sqrt{1}$

6. $\sqrt{4}$

7. $-\sqrt{25}$

8. $\sqrt{36} - \sqrt{49}$

9. $\sqrt{121}$

10. $\sqrt{64} + \sqrt{4}$

11. $-\sqrt{36} + \sqrt{9}$

12. $\sqrt{49} - \sqrt{25}$

CHALLENGE: Continue the list of square numbers above by writing the next three square numbers and their square roots.