WORKING WITH TWO-COLOR COUNTERS

ENGAGING ACTIVITIES FOR FRACTIONS, EQUATIONS, PROBABILITY AND REASONING

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Dedication

This book is dedicated to all those mathematics teachers who may be struggling to make mathematics more understandable, enjoyable, and interesting for their students using a hands-on manipulative approach.

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Introduction

The idea of Two-Color Counters is not a new concept. It actually began many years ago with teachers spray-painting lima beans, one color on one side and a different color on the opposite side. This makeshift procedure continued for several years, but it was not without drawbacks. Some beans chipped easily and were not the best models, and in the more humid climates the beans actually sprouted during the summer!

But teachers, being the creative individuals they are, found a solution to such problems. In 1982, a mathematics specialist in Wake County, North Carolina, named Kay Kemp (Ewell) came up with the brilliant idea of having circular plastic chips made, with one color on one side and another color on the flip side. Various color combinations were tried, and finally Kay decided that the red and yellow combination made a suitable model, red to correspond to the concept of "in the hole or negative" and yellow just because it is a pleasant color. A popular educational manipulative company picked up on the idea, and "two-color counters" as we know them today were born.

It didn't take long for educators to discover that many elementary mathematics concepts could be modeled nicely with two-color counters. Later, the use of two-color counters expanded into middle and secondary schools, where they lent themselves well to the modeling of more advanced concepts.

Today, two-color counters remain popular as models for a variety of concepts. The purpose of this book is to provide middle school teachers with proven techniques and ideas to help their students see certain abstract mathematical concepts manipulatively and thus to better understand them. I have personally used every one of these ideas in mathematics classes and found them to be extremely helpful.

The Structure of This Book

There are four basic parts to the lessons in this book:

1. **Introduction** of topics, ideas, and challenges.
2. **Main Ideas:** The purpose of these inserts is to provide information for those students who may be doing a lesson as independent work rather than as part of a class presentation. These "main ideas" needs to be presented and assimilated for the student to completely understand the lesson. The Main Idea boxes can serve as a heads-up to teachers, who sometimes assume this information is already known by the student and therefore omit it.

3. **Examples** are often provided so students can better grasp the topics being presented.

4. **Activities** for discovery, mastery, and discussions are provided.

The Science of Mathematics

A word about the methodology used in this book: Mathematics is often defined as the "science of patterns and relationships." Like scientific studies, the exploration of mathematics usually contains four facets:

1. Experimentation, explorations, investigations (all basically the same)
2. Observations
3. Drawing conclusions
4. Verification (proofs)

As you go through the activities in the following chapters, you will see that the approaches used incorporate these four facets as much as possible.

The Role of Discovery

Finally, I would be remiss if I did not say a word about the role discovery plays in mathematics learning. Research has shown that discovery is a powerful tool in helping students to learn, internalize, and remember important mathematical concepts.

Piaget once said, "Algorithms are generalized expressions of a rule or a systematic representation of a mathematical pattern. If students can experience the rewards and motivation of their own discoveries leading up to an algorithm, mathematics can come alive for them and they will want to further study mathematics. If not, mathematics can become a laborious task of memorizing a set of meaningless rules."

Even if the students cannot discover the patterns themselves, at least they can see the ideas through concrete presentations. This author strongly agrees with Piaget's statement and has written this book to illustrate activities that can easily lead to discovery and the joy it brings with it.

—Dr. Carl Seltzer

1.1

Fractions as Ratios of Equal Parts to a Whole

Main Ideas

Defining Fractions

A *fraction* is a number in the form of a ratio of equal parts to the whole, and a *ratio* is a comparison of two things.

Defining the "Whole"

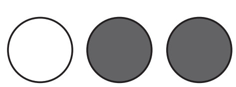
When dealing with fractions and manipulatives, the idea of the whole is very important. One-half does not always equal one-half unless we are comparing the same whole (the same 1). One-half of a dozen eggs does not equal one-half of a pizza or one-half of six eggs. As we study fractions with two-color counters, we will always need to define the "whole."

Defining Two-Color Counters

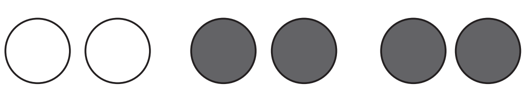
We will also need to define the counters. In this book, yellow counters are represented by ○ and red counters are represented by ●. Typically, yellow counters represent positive numbers and red counters represent negative numbers, but this isn't always the case. It depends on the mathematics being studied.

Example

If you were asked to model a fraction to show $\frac{2}{3}$ of a set, you could show the following:



Red = $\frac{2}{3}$, or



Red $= \frac{4}{6} = \frac{2}{3}$.

In this example, note that the red counters represent the numerator of the fraction being modeled, while the total number of counters represent the denominator.

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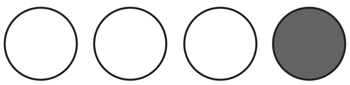
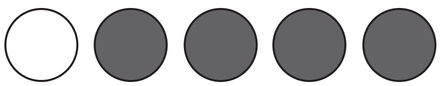
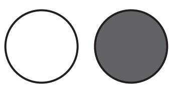
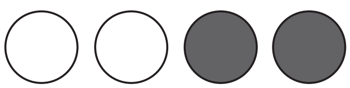
Fractions as Ratios of Equal Parts to a Whole

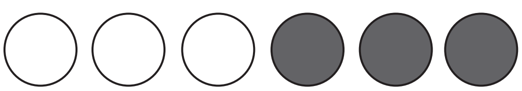
Write the fraction that corresponds to the number of *yellow* counters. Duplicate each model with your two-color counters.

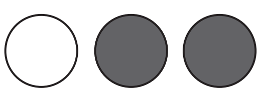
1.  __________

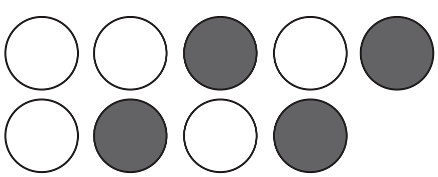
2.  __________

3.  __________

4.  __________

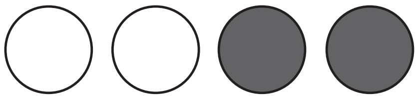
5.  __________

6.  __________

7.  __________

1.2

Equivalent Fractions

Example:  $= \frac{2}{4} = ?$

Suppose we want to find an equivalent fraction for $\frac{2}{4}$. First we need to define the whole. Let's use 4 two-color counters to represent "1"—the whole. If we want to reduce $\frac{2}{4}$ to an equivalent fraction, we will make two of the four counters red and the other two yellow. Then we can see that $\frac{2}{4}$ also equals one of the two groups, $\frac{1}{2}$.

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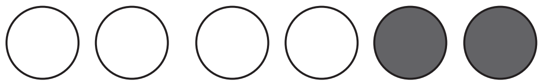
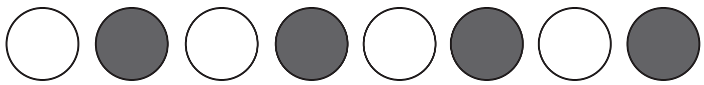
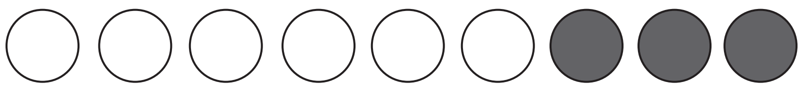
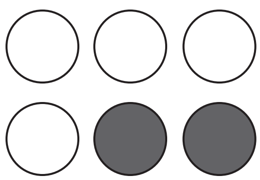
Practice Finding Equivalent Fractions

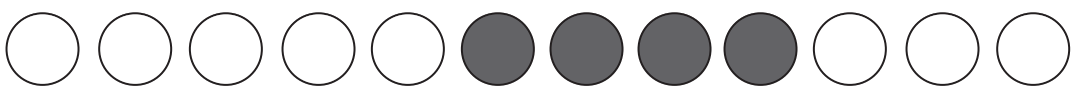
Find an equivalent fraction for each of the following.

1.  Y = __________

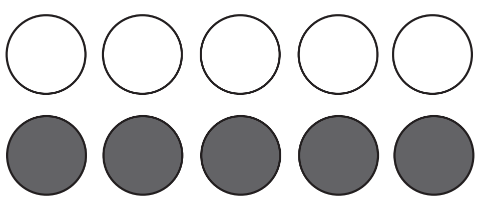
2.  R = __________

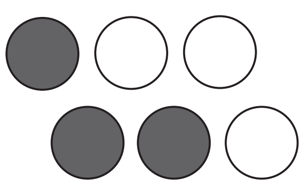
3.  R = __________

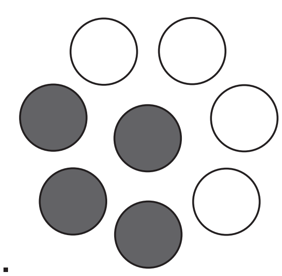
4.  Y = __________

5. 

Y = __________

6.  Y = __________

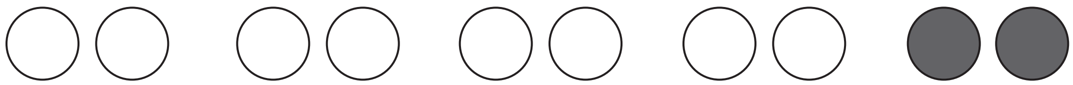
7.  R = __________

8.  R = __________

Name: ______________________ Date: ______________________

Practice Finding Equivalent Fractions

Demonstrate the following fractions and their equivalence with two-color counters. Use yellow counters to represent the denominator. Draw your answers to show the equivalence. (The first one has been done for you.)

1. $\frac{8}{10} = \frac{4}{5}$ 

2. $\frac{10}{12} = \frac{5}{6}$ ______________________

3. $\frac{1}{2} = \frac{2}{4}$ ______________________

4. $\frac{3}{4} = \frac{6}{8}$ ______________________

5. $\frac{6}{9} = \frac{2}{3}$ ______________________

6. $\frac{3}{5} = \frac{6}{10}$ ______________________

7. $\frac{1}{9} = \frac{2}{18}$ ______________________

8. $\frac{4}{12} = \frac{1}{3}$ ______________________

9. $\frac{1}{3} = \frac{3}{9}$ ______________________