

Good Questions for MATH TEACHING

Why Ask
Them
and What
to Ask
Grades
5–8



Lainie Schuster

Nancy Canavan Anderson



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PART ONE



The Practice of Good Questioning

Questions may be one of the most powerful technologies invented by humans. Even though they require no batteries and need not be plugged into the wall, they are tools which help us make up our minds, solve problems, and make decisions.

—Jamie McKenzie

“Good” questions can set the stage for meaningful classroom discussion and learning. When we ask good questions in math class, we invite our students to think, to understand, and to share a mathematical journey with their classmates and teachers alike. Students are no longer passive receivers of information when asked questions that challenge their understandings and convictions about mathematics. They become active and engaged in the construction of their own mathematical understanding and knowledge.

As we work to emphasize problem solving, application of concepts and procedures, and the development of a variety of thinking skills in our mathematics curricula, it becomes vital that we pay increased attention to the improvement of our questioning techniques in mathematics lessons, according to Peter Sullivan and Pat Lilburn, authors of *Good Questions for Math Teaching: Why Ask Them and What to Ask, K–6* (2002) and upon whose work this book is modeled. As teachers of mathematics, we want our students not only to understand *what* they think but also to be able to articulate *how* they arrived at those understandings. Developing productive questions can help focus learning on the process of thinking while attending to the study of content (Dantonio and Beisenherz 2001, 60). Good questions created and posed by teachers ultimately become powerful tools for student learning.



What Are Good Questions?

Careful, intentional, and mindful questioning is one of the most powerful tools a skillful teacher possesses (Costa and Kallick 2000, 34). So what do good questions “look” like?

- They help students *make sense* of the mathematics.
- They are *open-ended*, whether in answer or approach. There may be multiple answers or multiple approaches.
- They empower students to *unravel their misconceptions*.
- They not only *require the application of facts and procedures* but encourage students to *make connections and generalizations*.
- They are *accessible to all students* in their language and offer an entry point for all students.
- Their answers *lead students to wonder more about a topic* and to perhaps construct new questions themselves as they investigate this newly found interest.



How Are Good Questions Created?

While teacher guides provide direction and offer questions to ask, it is the teacher who must craft good questions to guide students to new learning and understanding.

Creating good questions relies heavily on the destination we have in mind. What do we expect our students to be able to do, say, or understand by the end of the lesson? Beginning with the end in mind requires us to start with a clear understanding of desired knowledge, learning, and outcomes. When we think about questions that we might ask our students, it is helpful to consider these issues:

- the mathematical goals of the lesson
- the misconceptions students may have
- the connections we'd like students to make between lesson goals and previously covered concepts and/or procedures
- assessment of understanding

The questions in this book may help you begin a unit of study. They may help students make connections between new material and previously covered concepts and procedures. They may offer opportunities for students to confront faulty thinking or fragile understanding. There are also those questions that encourage students to wonder and that push *them* to pose new questions as they seek greater understanding.



What Are the Teacher's Responsibilities in Presenting Good Questions?

When presenting good questions, it is essential for teachers to

- understand the mathematics embedded in the question
- present the question clearly using accessible mathematical language
- set clear and reasonable expectations for student work
- allow for individual approaches, methods, and/or answers
- add variety or more data to a question to ensure accessibility for all students
- make good use of concrete materials
- allow ample time for discovery and consolidation of answers, strategies, and the discovered mathematics

Someone somewhere said, “It’s all in the presentation,” and correct he or she was. In order to engage students and make the process of answering a question meaningful, we need to be mindful of our presentation and expectations. It is important for teachers to work through a question before presenting it to students—to solve it and think through it themselves. This is of particular importance with open questions since these questions have many different approaches and/or answers. Try to think of more than just one response. Doing so will help you anticipate and then react to the variety of responses you will hear from your students. Undoubtedly, students will think of things you did not! In addition to working through the question, think about the following:

- Do I have the necessary materials (for example, graph paper, chart paper, manipulatives)? If not, where can I get them?
- What misconceptions or difficulties might my students have with the language, concepts, or directions?

- What follow-up questions can I ask that will readdress or redirect misconceptions or difficulties?
- How much time will students need to answer the question?

Working through good mathematics takes time. An understanding of the mathematics involved in the question being presented will help you determine the amount of time a question may take to answer and process. It is important, however, that you give ample time to discussing both students' answers and the reasoning behind those answers. Students need enough time to fully develop a thought or conjecture as answers are discussed. Because we are so pressed for time in our classrooms, we often give more time to question *answering* than to *processing*. Having students share and discuss answers and strategies can be as important to learning as the act of answering. There may be those times when a question and its answers take on a life of their own—and a full math period! Please allow that to happen once in a while. Students will take greater ownership of their thinking and learning when they realize that it is *they* who are determining the course of study.

4 What Classroom Conditions Are Necessary to Support Good Answers?

To question well is to teach well. Good questioning practice requires classroom routines that may take time and patience to establish but will be well worth the time and effort. It should also be noted that not only can we enrich our mathematics instruction by employing these practices, but adapting similar routines in other disciplines may improve student understanding and engagement in those areas as well.



Creating a Safe Environment

The teaching and learning of mathematics can be an emotionally charged practice. Teachers and students come to the table with different sets of expectations and proficiencies. When good questions are presented in a safe classroom environment, learning, communication, and enthusiasm can quickly ensue. Students are taking risks when they answer open-ended questions. Because there is often not just one correct answer or process, students need to be encouraged to keep open minds and support the thinking of their peers. Disagreements will occur . . . and we hope they do! Some of our most powerful learning occurs as we try to prove or disprove what appears to be faulty thinking. Students need to realize, however, that they may disagree with another's *ideas*, which is quite different from disagreeing with the person himself. Students are to treat each other civilly and with respect. This may take vigilant monitoring on the part of the teacher as a mathematically safe environment is developed and maintained.



A Place to Begin

Considering how students may begin the process of answering a question can be an important issue in many classrooms. You may need to make adjustments in the language and/or directions of the posed question in order to support struggling students and even those students who always seem to get it quickly.



Waiting

Purposeful, consistent, and patient wait time will ultimately increase engagement and active participation. If our goal is to elicit mindful, insightful, and mathematically sound answers, then we need to give our students time to think and formulate their conjectures and answers before opening up classroom discussions. We live in a world of immediacy, so asking students to use the silence of wait time to think and formulate their answers before starting a conversation can be difficult for everyone involved. Employing the five-second rule after each time you ask a question or call on a student will help establish this important practice.



Discussing Answers

Once students have had enough time to formulate an answer to a question, they benefit from hearing one another's answers. First, ask students to talk in a small-group format—either in pairs or with table groups. Multiple conversations can occur, and the teacher should circulate and take notice of generated insights, strategies, partial understandings, and misconceptions. These small-group conversations often give students the time and opportunity to rehearse their thinking. Students may be more willing to share their reasoning and ideas with the whole class once they have first had a chance to practice with a small group. Students need to be encouraged and reminded to keep written records of their thinking together—calculations, charts, diagrams, and/or pictures. This documentation will help them support and defend their thinking and answers.

When it is time for the talk to move to a whole-class discussion, the teacher can guide the conversation by keeping the focus on the students' thinking. Students need to be encouraged to support, add to, and even disagree with the strategies, insights, and answers of fellow classmates. They also need to address one another, not merely the teacher. This may be a new approach for both

teachers and students. Class discussions offer opportunities for students to achieve understanding by processing information, applying reasoning, hearing ideas from others, and connecting new thinking to what they already know (Chapin, O'Connor, and Anderson 2003). The following list offers a generic set of questions that may help guide and facilitate discussions of students' answers:

- Why do you think that?
- How did you know to try that strategy?
- How do you know you have an answer?
- Will this work with every number? Every similar situation?
- When will this strategy not work? Can you give a counterexample?
- Who has a different strategy?
- How is your answer like or different from another student's?
- Can you repeat your classmate's ideas in your own words?
- Do you agree or disagree with your classmate's idea? Why?