Contents

Foreword by Carmen Whitman, Series Editor xi
How to Use This Resource xiii
Correlations to the Common Core State Standards xvii

SECTION I Qualitative Graphs

Overview 2

LESSON 1.1 Prices in Washington 4
Given a sentence, choose a graph

LESSON 1.2 Value of a Portrait 8
Given a graph, choose a sentence; draw graphs

LESSON 1.3 Temperature for Tuesday 11
Interpret and draw graphs; incorporate units of measurement

LESSON 1.4 Population in Terms of Time 14
Learn different ways to increase

LESSON 1.5 Weather in Stony Creek, Rockville, and Boulder 19
Learn different ways to decrease

LESSON 1.6 Owning a Home 24
Recognize and draw increasing functions with specified rates of change

LESSON 1.7 Cars and Walking to School 27
Recognize and draw decreasing functions with specified rates of change

LESSON 1.8 Distance and Population 30
Recognize and draw graphs with specified rates of change

LESSON 1.9 An Antique, a Jewel, and a Car 33
Compare and contrast values; create multiple graphs on one set of axes

LESSON 1.10 Distance to a Bench 37
Compare and contrast linear graphs

LESSON 1.11 At the Beach 42
Compare and contrast distance-versus-time graphs
Further Practice

FURTHER PRACTICE 1.A  Value, Population, and Distance over Time  45
Practice writing sentences describing graphs
(Can be done any time after Lesson 1.2)

FURTHER PRACTICE 1.B  Your Choice and Animals at the Zoo  48
Practice rates of change with increasing functions
(Can be done any time after Lesson 1.4)

FURTHER PRACTICE 1.C  More Your Choice  50
Practice with varying rates of change
(Can be done any time after Lesson 1.8)

SECTION II  Quantitative Graphs

Overview  54

LESSON 2.1  Numbers on a Line  56
Estimate points and tick marks along axes given certain values

LESSON 2.2  Height of a Person  60
Estimate scales along axes given height as a function of time

LESSON 2.3  Value of a Car  65
Read and interpret a quantitative graph of a decreasing function

LESSON 2.4  Rising Real Estate Prices  68
Interpret a quantitative graph of an increasing function with a decreasing rate of change

LESSON 2.5  Perimeter of a Square  71
Use the graph of the linear function \(y = 4x\)

LESSON 2.6  Area of a Square  76
Use the graph of the quadratic function \(y = x^2\)

LESSON 2.7  A Rumor in the Classroom  81
Interpret a graph with a point of inflection

LESSON 2.8  Using a Motion Detector  85
Interpret a quantitative graph of a decreasing function with a decreasing rate of change

LESSON 2.9  Mark’s Distance from the Cafeteria  90
Work with a distance-versus-time graph

LESSON 2.10  Value of a Boat  95
Work with a decreasing function with an increasing rate of change

LESSON 2.11  Rocket in the Air  100
Work with a quadratic function that models the height of a rocket in terms of time
LESSON 2.12  Beehive Rocket  103
Draw the path of a rocket

SECTION III  Including Tables with Graphs
Overview  108

LESSON 3.1  Spread of a Virus  110
Lesson similar to Section II, table introduced at the end

LESSON 3.2  Distance from a Dock  114
Table and graph for a constant function

LESSON 3.3  Distance from a Motion Detector  117
Given a table, answer questions, and produce a graph

LESSON 3.4  Value of an Antique  120
Given a table, consider rate of change and graph

LESSON 3.5  Value of a Motorcycle  124
Given a linear relationship, produce a table and a graph

LESSON 3.6  Distance from a House  129
Given a graph, choose a table; given a table, choose a sentence

LESSON 3.7  The Height of a Projectile  133
Given a quadratic function, answer questions, and produce a table

LESSON 3.8  Throwing a Grapefruit!  137
Further work with quadratic functions

FURTHER PRACTICE 3.A  Your Choice  141
Create your own example (Can be done any time after Lesson 3.5)

SECTION IV  Adding in Equations
Overview  144

LESSON 4.1  Value of a Computer  146
Work from description, to table, to graph, to equation

LESSON 4.2  Purchase of a Jewel  150
Work from graph, to description, to table, to equation for an increasing linear function

LESSON 4.3  Distance from a Building  153
Work from graph, to description, to table, to equation for a decreasing linear function

LESSON 4.4  Stan Is Waiting  157
Work with a constant function
LESSON 4.5  Mary and the Pool  160
Put it all together with an increasing linear function

LESSON 4.6  The Cost of Pasta  163
Work from description, to table, to graph, to equation for a cost function

LESSON 4.7  Carl at the Gate  166
Put it all together with a decreasing linear function

LESSON 4.8  A Rocket in the Air  169
Work from equation, to description, to graph, to table for a quadratic function

LESSON 4.9  Another Rocket in the Air  174
Understand quadratic functions

Index  179
Overview

In this lesson, students are introduced to the concept of a point of inflection.

Teaching the Lesson

In this lesson students are introduced to the concept of a point of inflection. In addition, they again identify the units of measurement, locate values, and translate points into statements based on the application.

Homework and Assessment

For homework after finishing the recording sheet, ask students to solve these problems:

1. Suppose a class has thirty people in it, and three people start a rumor. What would that graph look like? Compare and contrast this graph with the one shown on the recording sheet.

2. A company president claimed at first that the company’s debt was increasing at an increasing rate but now claims the debt is increasing at a decreasing rate. What would that graph look like compared to the graph on the recording sheet?

Answers to Lesson 2.7 Student Recording Sheet and Homework

STUDENT RECORDING SHEET

1.
   a. The graph shows the relationship between time and people who have heard the rumor.
   b. The units of measurement along the x-axis are minutes.
   c. The units of measurement along the y-axis are numbers of people.
   d. Two people start the rumor.
e. There are twenty-four people in the class. (Recall that the introduction to the problem says that everyone has heard the rumor within ten minutes.)

f. The function is increasing for the first 7 minutes and then remains constant. This is shown by the curve’s rising and then remaining at the same level as we move along the x-axis from left to right. (A typical student answer to this question is “It goes up.” Encourage students to write a more complete answer, such as, “As time passes, the y value is going up and then remaining at the same level” or “As we move from left to right along the x-axis, the curve is rising and then remaining at the same level.”)

g. The point of inflection is at about (2.5,12).

h. At the point of inflection, twelve people have heard the rumor.

i. At the point of inflection, it has been about two and a half minutes since the rumor started.

j. After four and a half minutes, twenty-two people have heard the rumor.

k. The y-intercept is the number of people who first heard the rumor.

**HOMEWORK**

1. The graph would have a y-intercept of 3, not 2. It would level off at 30, not 24.

2. The graph showing the company’s debt over time will also look like the graph on the recording sheet.

---

**Extend the Learning:** The population of a species of animals on an island often has the shape of the “rumor” graph. Can you explain why? Suppose the population starts with twenty-five animals and levels off at 300 after six years. Assume the point of inflection is at (2,150). Draw the graph, and use it as the basis for your explanation.
1. A rumor is spreading quickly among the students in a classroom, and within ten minutes, everyone has heard it. The graph below shows the number of people who have heard the rumor in relation to the time that has passed since the rumor was started.

![Graph showing the number of people who have heard the rumor in relation to time.](image)

a. This graph shows the relationship between what two quantities?

b. What are the units of measurement along the $x$-axis?

c. What are the units of measurement along the $y$-axis?

d. How many people start the rumor?

(continued)
e. How many people are in the class?

f. The relationship between the quantities on this graph is called a function, because each x value leads to one and only one y value. Is the function decreasing? How can you tell?

g. The function changes more and more quickly and then more and more slowly. The point at which it shifts from faster and faster to slower and slower is called the point of inflection. Find and mark the point of inflection on the curve.

h. At the point of inflection, how many people have heard the rumor?

i. At the point of inflection, how long has it been since the rumor started?

j. A point is marked on the curve. Write a sentence about the rumor based on the information provided by the coordinates of the point.

k. What does the y-intercept mean in terms of this problem?
Correlations to the Common Core State Standards

In this correlation you will find some lessons that span several grade levels. A lesson can be used as an introduction to different types of representations, developing the student’s understanding of the general nature of graphs, tables, or equations. It can be used later in the student’s career to refresh their understanding or to assess their knowledge of a topic as they proceed with the development of more complex problems. Lessons, homework suggestions, and extensions that encompass the lessons were all considered when creating the correlation.

Modeling is best interpreted not as a collection of isolated topics but in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★). The star symbol sometimes appears on the heading for a group of standards; in that case, it should be understood to apply to all standards in that group (corestandards.org/assets/CCSSI_Math Standards.pdf).

### Grade 6

**Domain: Ratios and Proportional Relationships—6.RP**

Understand ratio concepts and use ratio reasoning to solve problems.

3. Use ratio and rate reasoning to solve real-world and mathematical problems (e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations).
   - Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
   - Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

### The Number System — 6.NS

Apply and extend previous understandings of numbers to the system of rational numbers.

6. Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates.
   - Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes.
   - Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

### Expressions and Equations — 6.EE

Reason about and solve one-variable equations and inequalities.

5. Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
<table>
<thead>
<tr>
<th>Grade 6</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.</td>
</tr>
<tr>
<td>7.</td>
<td>Solve real-world and mathematical problems by writing and solving equations of the form ( x + p = q ) and ( px = q ) for cases in which ( p, q ) and ( x ) are all nonnegative rational numbers.</td>
</tr>
<tr>
<td></td>
<td>Represent and analyze quantitative relationships between dependent and independent variables.</td>
</tr>
<tr>
<td>9.</td>
<td>Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation ( d = 65t ) to represent the relationship between distance and time.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Grade 7</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratios and Proportional Relationships — 7.RP</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Analyze proportional relationships and use them to solve real-world and mathematical problems.</td>
</tr>
<tr>
<td>2.</td>
<td>Recognize and represent proportional relationships between quantities.</td>
</tr>
<tr>
<td>a.</td>
<td>Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin).</td>
</tr>
<tr>
<td>b.</td>
<td>Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.</td>
</tr>
<tr>
<td>c.</td>
<td>Represent proportional relationships by equations. For example, if total cost ( t ) is proportional to the number ( n ) of items purchased at a constant price ( p ), the relationship between the total cost and the number of items can be expressed as ( t = pn ).</td>
</tr>
<tr>
<td>d.</td>
<td>Explain what a point ((x,y)) on the graph of a proportional relationship means in terms of the situation, with special attention to the points ((0,0)) and ((1,r)) where ( r ) is the unit rate.</td>
</tr>
<tr>
<td><strong>Expressions and Equations — 7.EE</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Solve real-life and mathematical problems using numerical and algebraic expressions and equations.</td>
</tr>
<tr>
<td>4.</td>
<td>Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.</td>
</tr>
<tr>
<td>a.</td>
<td>Solve word problems leading to equations of the form ( px + q = r ) and ( p(x + q) = r ), where ( p, q, ) and ( r ) are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?</td>
</tr>
</tbody>
</table>
## Grade 8

### Expressions and Equations — 8.EE

Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

   Lessons 1.10, 1.11, 3.5, and 4.6

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.

   Lessons 1.10 and 1.11
   Lessons 2.8 and 2.9

### Functions — 8.F

Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

   Note: Function notation is not required in Grade 8.

2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

   Lessons 3.1 and 3.6

3. Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function \( A = s^2 \) giving the area of a square as a function of its side length is not linear because its graph contains the points \((1,1), (2,4)\) and \((3,9)\), which are not on a straight line.

   Lessons 1.8, 2.4, 2.6, 4.5, 4.7, 4.8, and 4.9

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x,y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

   Lessons 1.1, 1.2, 1.3, 1.4, 2.3, 2.7, 3.2, 4.3, 4.4, 4.5, and 4.7

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

   Lessons 1.3, 1.4, 1.5, 1.6, 1.7, 1.8, 1.9, 1.10, 1.8, 2.3, 2.8, 2.9, 3.6, 4.3, 4.4, 4.5, 4.7, 4.8, and 4.9
### Algebra

#### Creating Equations — A-CED

Create equations that describe numbers or relationships.

| 1. | Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. | Lessons 4.2, 4.6, 4.7, 4.8, and 4.9 |
| 2. | Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. | Lessons: 2.9, 4.5, 4.6, 4.7, 4.8, and 4.9 |

#### Reasoning with Equations and Inequalities — A-REI

Solve equations and inequalities in one variable.

| 3. | Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. | Lessons 4.2 and 4.4 |

Represent and solve equations and inequalities graphically.

| 10. | Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). | Lessons 2.3, 2.4, 2.5, 2.6, 2.7, 2.8, 3.3, 3.4, 3.7, 4.8, and 4.9 |

### Functions

#### Interpreting Functions — F-IF

Understand the concept of a function and use function notation.

| 1. | Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If \( f \) is a function and \( x \) is an element of its domain, then \( f(x) \) denotes the output of \( f \) corresponding to the input \( x \). The graph of \( f \) is the graph of the equation \( y = f(x) \). | All Lessons |

Interpret functions that arise in applications in terms of the context.

| 4. | For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. | Lessons 1.3, 1.4, 1.5, 1.6, 1.7, 1.10, 2.2, 2.4, 3.8, 4.5, 4.8, and 4.9 |

Analyze functions using different representations.

| 7. | Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. | Lessons 2.9, 2.11, 2.12, 3.7, 4.5, 4.8, and 4.9 |
| a. | Graph linear and quadratic functions and show intercepts, maxima, and minima. | |

#### Building Functions — F-BF

Build a function that models a relationship between two quantities.

| 1. | Write a function that describes a relationship between two quantities. | Lessons 1.3, 1.5, and 2.10 |
| b. | Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. | |
### Functions

<table>
<thead>
<tr>
<th>Linear, Quadratic, and Exponential Models ★— F-LE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Construct and compare linear, quadratic, and exponential models and solve problems</td>
</tr>
<tr>
<td>1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</td>
</tr>
<tr>
<td>b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.</td>
</tr>
<tr>
<td>Lessons 2.5, 2.7, 4.5, and 4.6</td>
</tr>
</tbody>
</table>